

## 6 Network model calibration studies

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This chapter details how calibration and estimation works in the realm of large networks, and this is mainly related to OD-estimation which is arguably the most important step before applying a simulation model (be it microscopic or macroscopic) to a certain study question.

### 6.1 Calibration of network models

Calibration variables associated with a simulation model are grouped based on the constituent model component, whose inputs and parameters are under consideration. In general, simulation model components are divided into two categories:

1. Demand models that estimate and predict OD trip flows and simulate travel behaviour parameters;
2. Supply models that capture traffic dynamics.

Hence, the problem of simulation model calibration involves estimation and calibration of all its components.

Generally, demand and supply models are calibrated separately. However, methodologies for joint demand-supply calibration have also been developed in recent time. The methods for sequential and simultaneous calibration of the demand and supply parameters using aggregate data are reviewed in the following sections.

### 6.2 Demand models

Demand parameters calibration involves estimation of travel behaviour parameters and OD demand flows. Travel behaviour parameters are presented by pre-trip departure time, mode and route choice, en-route response to information, which are generally estimated using disaggregate data that are not considered in the presented chapter. This class of parameters can be calibrated simultaneously with OD demand flows using aggregate data.

The review of demand model estimation techniques begins with discussion of OD demand matrices estimation and continues with presentation of the methods for joint estimation of OD demand flows and travel behaviour parameters.

#### 6.2.1 OD demand estimation

Many different approaches have been proposed in order to solve the OD estimation problem. They can be grouped into two main classes: direct estimation methods (surveys) and estimation methods using traffic counts. In general, the direct estimation methods are time and cost expensive. In recent years, increasing attention has been devoted to more effective methods of OD demand estimation using traffic counts.

OD estimation methods based on traffic counts can be classified using two factors: network performances and temporal demand dynamics. With respect to the network performances, a distinction is made between congested and uncongested networks. Estimated travel demands are also classified as steady-state (static) or time-dependent (dynamic) matrices.

## 6.2.2 Static OD demand estimation methods

### 6.2.2.1 Uncongested networks

OD demand estimation methods for uncongested networks assume that link and path travel times can be computed from the network model, even though link flows are not available.

The methods proposed in the literature for the static OD demand estimation problem on uncongested networks include information minimization (Van Zuylen, 1979), entropy maximization (Van Zuylen and Willumsen, 1980; Brenninger-Gothe et al., 1989; Lam and Lo, 1991), maximum likelihood estimation - MLE (Spiess, 1987; Cascetta and Nguyen, 1988; Lo et al., 1996; Hazelton, 2000), generalized least square - GLS (Cascetta, 1984; Bell, 1991) and Bayesian inference (Maher, 1983).

Following Cascetta and Nguyen (1988) and Cascetta (2001), classical estimators provide for a Maximum Likelihood estimate  $\mathbf{d}_{ML}$  of the demand vector by maximizing the probability (likelihood) of observing both O-D sampling survey data and link counts (under the usually acceptable assumption that these two probabilities are independent), yielding:

$$\mathbf{d}_{ML} = \arg \max_{\mathbf{x} \geq 0} \left[ \ln L(\hat{\mathbf{d}}|\mathbf{x}) + \ln L(\hat{\mathbf{f}}|\mathbf{x}) \right] \quad (6.1)$$

where  $\mathbf{x}$  is the variable demand,  $\hat{\mathbf{d}}$  is the demand by sample and  $\hat{\mathbf{f}}$  the vector of link counts. Log-likelihood functions in equation (6.1) are specified on the basis of hypotheses on the probability distribution of demand counts  $\hat{\mathbf{d}}$  and traffic counts  $\hat{\mathbf{f}}$  respectively, conditional on the demand vector  $\mathbf{x}$ . Normally, traffic counts can be assumed as independently distributed as Poisson random variables, or following a Multivariate Normal random variable, while the statistical distribution of O-D demand counts depends on the sampling strategy. Generalized Least Squares (GLS) demand estimation  $\mathbf{d}_{GLS}$  provides an estimate of the O-D demand flow, starting from a system of linear stochastic equations, leading to the following optimization problem:

$$\mathbf{d}_{GLS} = \arg \min_{\mathbf{x} \geq 0} \left\{ \frac{1}{2} (\mathbf{x} - \hat{\mathbf{d}})^T \mathbf{Z}^{-1} (\mathbf{x} - \hat{\mathbf{d}}) + \frac{1}{2} (\hat{\mathbf{f}} - \mathbf{M}_f \mathbf{x})^T \mathbf{W}^{-1} (\hat{\mathbf{f}} - \mathbf{M}_f \mathbf{x}) \right\} \quad (6.2)$$

where  $\mathbf{M}_f$  is the sub-matrix of the assignment matrix related to links with available traffic counts and  $\mathbf{Z}$  and  $\mathbf{W}$  the covariance matrices related to the sampling error underlying the demand estimation and the measurement/assignment errors respectively.

Bayesian methods estimate unknown parameters by combining experimental information (traffic counts in this case) with non-experimental information (*a priori* or “subjective” expectations on O-D demand, e.g. coming from an out-of-date estimation or from a model system), by maximizing the logarithm of the *a posteriori* probability:

$$\mathbf{d}_B = \arg \max_{\mathbf{x} \geq 0} \left[ \ln g(\mathbf{x}|\mathbf{d}^*) + \ln L(\hat{\mathbf{f}}|\mathbf{x}) \right] \quad (6.3)$$

where  $g(\mathbf{x}|\mathbf{d}^*)$  expresses the distribution of subjective probability attributed to the unknown vector given the *a priori* estimate  $\mathbf{d}^*$  and  $L(\hat{\mathbf{f}}|\mathbf{x})$  expresses the probability of observing the vector of traffic counts  $\hat{\mathbf{f}}$  conditional on the unknown demand vector  $\mathbf{x}$ . Again, the detailed specification of a Bayesian estimator depends on the assumptions made about the probability functions  $g(\mathbf{x}|\mathbf{d}^*)$  and  $L(\hat{\mathbf{f}}|\mathbf{x})$ . Normally, the unknown demand vector can be assumed to follow a multinomial random variable (in this case  $\ln g(\mathbf{x}|\mathbf{d}^*)$  becomes the entropy function of the unknown vector  $\mathbf{x}$ ), a Poisson random variable (in this case  $\ln g(\mathbf{x}|\mathbf{d}^*)$  becomes the information function of the unknown vector  $\mathbf{x}$ ), or a Multivariate Normal random variable. Notably, it should be noted that approaches (6.1)-(6.3) result in objective functions of identical form if all the relevant distributions are assumed to be multivariate normal.

### 6.2.2.2 Congested networks

In the congested network case, travel times, path choice fractions and assignment fractions depend on link traffic flows. Generally, neither travel times nor flows are known on all links. Instead, they have to be computed by applying an assignment model, the result of which in turn depends on the assigned OD matrix. The problem of

the circular dependence between OD matrix estimation and traffic flow assignment on congested networks has been studied by different authors.

A bi-level programming approach is one of the possible solutions to ensure the interdependency between OD demand matrix and traffic assignment. In the bi-level approach, the upper-level problem uses one of the statistical techniques proposed earlier (e.g., maximum entropy, GLS, MLE) to select the most appropriate OD matrix. The lower-level problem, using deterministic or stochastic user equilibrium models (Sheffi, 1985), endogenously determines route choice proportions that are compatible with the estimated OD flows.

Several solution methods were proposed to handle the bi-level program. Fisk (1988) combined the entropy maximum model with equilibrium conditions to construct the bi-level programming problem and used a variational inequality formulation to solve it. Heuristic iterative algorithms for the bi-level estimation problem solution were proposed by Spiess (1990), Yang et al. (1992), Yang (1995), Yang and Yagar (1995) and Maher and Zhang (1999). The models proposed by Cascetta and Postorino (2001), Maher et al. (2001) and Yang et al. (2001) differ from the previous models in that they assume a stochastic user equilibrium.

Florian and Chen (1992, 1995) reformulated the bi-level problem into a single level equilibrium-based OD demand estimation problem using the concept of marginal function. However, most of these methods are acceptable only for small and medium scale networks. Lundgren and Peterson (2008) extended the earlier proposed methods (Spiess, 1990; Maher and Zhang, 1999) to adapt them for large scale networks. Kim et al. (2001) and Al-Bataineh and Kaysi (2006) used genetic algorithms as an alternative approach for solution of the bi-level OD estimation problem.

Cascetta and Postorino (2001) formulated the general static OD estimation problem for congested networks as a fixed point problem. To solve the fixed-point OD estimation problem and obtain consistent OD flows iterative schemes, based on the method of successive averages (MSA), were applied. The sequential GLS-based OD estimator was used to generate updated flows in each iteration.

Lo et al. (1996) introduced an explicit representation of the stochastic nature of observed flows, eventually generalized by Vardi (1996); Lo et al. (1999) describe an optimization method for the application of this approach to large-scale networks. Hazelton (2000) proposes a method which can also make use only of link counts, but it requires explicit path enumeration and is therefore, in practice, extremely time-consuming for large networks. Finally, as pointed out by Hazelton (2003), a promising research development entails considering time-series link counts (i.e. referred to several days) as a key aspect for improving the reliability of O-D matrix estimation: he took into account the covariance matrix of link count observations taken on several days within the estimation procedure, showing its reliability on very small test networks.

### *6.2.3 Dynamic OD demand estimation methods*

Static OD estimation models calculate a single matrix of mean OD demand values using the average traffic counts collected during a relatively long period. The time-varying nature of link flows and OD demands is ignored in these matrices, disregarding the departure time offsets between vehicles counted at some link, affecting their applicability in dynamic traffic management problems.

In recent years, different dynamic OD demand estimation models have been proposed to overcome the limitations of the static models. Dynamic OD estimation methods can be categorized into on-line and off-line approaches. Off-line methods simultaneously estimate demand and supply model parameters. The on-line methods jointly update in real-time the demand and supply parameter values estimated during the off-line step to better reflect prevailing conditions (Antoniou et al., 2009).

#### *6.2.3.1 On-line estimation methods*

Okutani (1987) developed the first on-line OD estimation model suitable for general networks. An autoregressive model is used to describe the evolution of OD flows over time. A Kalman filter was used to get optimal estimates of the state vector in each time interval. However, no information was provided on how the dynamic assignment fractions can be determined. Furthermore, the autoregressive process may not capture transitions of time-dependent demand patterns, such as that from peak to off-peak time periods.

Ashok and Ben-Akiva (1993) presented a Kalman filter based approach for on-line estimation and prediction. In order to improve Okutani's model, they introduce the notion of deviations of OD demand flows from historical

estimates. The proposed model was aimed to estimate and predict the within-day deviation of travel demands from its average day-to-day pattern. This model overcomes deficiencies of its several predecessors, but has some shortcomings. First, no attempt is made to capture errors in the assignment matrix. Secondly, the model requires augmenting the state for a given interval with states corresponding to several prior intervals. It increases the computational load of the problem; thereby making an on-line application of the model difficult computationally.

In subsequent work, Ashok (1996) indicated that the assignment matrix itself should be estimated, because it is computed from random variables such as link travel times. Two methods were suggested to obtain an assignment matrix. The first approach involves the use of a traffic simulator to load the current best OD flows onto the network. The assignment matrix then can be calculated through book-keeping of vehicle records at sensors. Another approach is computing the assignment fractions for a given sensor and OD pair by summing the product of crossing fractions and the corresponding path choice fractions across all paths.

Ashok and Ben-Akiva (2000) suggested another alternative approach, having redefined the state variables as the deviation of departure flows from each origin and the shares headed to each destination. Except the different form of transition equations, the approach has a similar framework as those they proposed earlier.

Ashok and Ben-Akiva (1993, 2000) reported encouraging results in their case studies and field tests, and indicated that their methods turned out to be robust. In the algorithm proposed by Bierlaire and Crittin (2004) the state described by Ashok and Ben-Akiva is combined with a general technique for solving large-scale least-square problems. This iterative algorithm is shown to be more efficient than the Kalman filter approach suggested by Ashok and Ben-Akiva.

#### 6.2.3.2 *Off-line estimation methods*

The off-line dynamic OD estimation problem was first formulated by Cascetta et al. (1993). The authors generalized the statistical framework proposed for the static problem and extended it to the dynamic OD estimation case. Estimates of dynamic OD demand flows are obtained by optimizing a two part objective function. The first part measures the difference between the estimated OD matrix for an interval and the historic estimate of the OD matrix for that interval. The second part measures the difference between measured link volumes and those predicted by the model when the estimated OD demand flows are assigned to the network.

The authors used the GLS method, combining traffic counts with other available information on OD flows such as earlier matrices and surveys. Two types of estimators - simultaneous and sequential - were proposed. The simultaneous estimator, designed for off-line applications, gives in one step the entire set of time-dependent OD demand vectors by using link traffic counts from all time intervals. The sequential estimator, suitable for on-line or large-scale applications, gives in each step the OD vector for a given time interval by using both previous OD estimates and the current and previous traffic counts. The simultaneous estimator, while statistically more efficient than its sequential approximation, entails significant computational overhead that precludes its application to large problems.

Cascetta et al. noted that link travel times can be obtained either from a traffic surveillance system or a day-to-day dynamic traffic assignment model. Either way, the implication is that the dynamic assignment matrix is exogenously determined so that it might be inconsistent with the estimated assignment mapping if the network is congested. This model was further developed by Tavana (2001) and Ashok and Ben-Akiva (2002).

Tavana and Mahmassani (2000) and Tavana (2001) proposed a bi-level optimization model and an iterative solution framework to estimate dynamic OD demand matrix. In the upper-level problem, the demand value is estimated by minimizing the sum of squared errors in traffic counts with the OD matrix entries. This optimization problem was solved, using the conventional GLS estimator and assuming that the link-flow proportions are constant. The dynamic user optimal conditions were treated as constraints. The resulting dynamic estimation problem was solved heuristically using an iterative optimization-assignment algorithm. Specifically, the dynamic assignment factors were updated iteratively using the dynamic user equilibrium solution.

Tavana's model was extended by Zhou and Mahmassani (2003). The authors proposed a multi-objective optimization framework to combine available historical static demand information and multi-day traffic link counts to estimate the variation in the traffic demand over multiple days.

Lindveld (2003) also proposed an iterative heuristic for the dynamic OD demand estimation problem. The problem was formulated as a bi-level optimization problem, where the estimate of dynamic OD matrix is found on

the upper level, using both traffic counts and a priori demand information, with dynamic traffic equilibrium constraints on the lower level. The proposed model is basically a time-dependent extension of the static model proposed by Maher and Zhang (1999), for the case of deterministic user equilibrium, and further developed by Maher et al. (2001) for stochastic user equilibrium. The method has been successfully implemented for a small test network with a corridor structure, i.e. a network with no route choices.

Sherali and Park (2001) proposed a constrained least squares (CLS) model but solved for path flows rather than OD demand flows. Following Casetta et al. (1993), it was assumed that the dynamic assignment matrix can be obtained externally. The proposed path generation algorithm begins with solving a restricted basic problem based on an initial choice of a set of OD paths, and then augments the basic problem iteratively with time-dependent shortest paths upon a time-space expanded network. The method will terminate and claim an optimum if no new time-dependent shortest paths can be found.

Most of the presented models are either developed for small networks, or they simplify the congestion effects on route choice and travel time (e.g. Cascetta, 1993; Sherali and Park, 2001). Lundgren et al. (2006) proposed a heuristic for estimation of dynamic OD demand matrices using traffic counts. Special interest in this work was given to the assignment matrix that depends on the demand in the case of congestion. The proposed method is an extension of the method presented by Maher and Zhang (1999) for deterministic static case and further developed by Maher et al. (2001) for stochastic static case. The authors used an iterative algorithm, based on the steepest descent method, to find a solution to the estimation problem. The directional derivatives of the assignment map for the current OD demand matrix are approximated with a difference quotient, which describes how a change of the OD demand matrix will induce a change of the link flows. However, this method was tested only on a relative small network and its effectiveness needs to be checked on larger networks.

#### 6.2.4 *Quasi Dynamic OD demand estimation methods*

One of the main problem in the OD demand estimation methods using traffic counts is the great unbalance between equation and unknowns. In other words, the equations available for the demand estimate (equal to the number of observed link flows) are generally much lower with respect to the number of unknowns (equal to the number of OD flows). This is obviously true both in the static and in the dynamic case, since passing from the static to the dynamic case, both equations and unknowns increase linearly proportionally to the number of time slices  $t$  considered. This circumstance, significantly influence the quality of the OD demand estimation, which, as demonstrated by Marzano et al (2009) through laboratory experiments, is strictly related to the quality of the a-priori OD estimation.

An effective method to obtain a good quality of the OD estimation also in the case of poor quality of the a-priori OD estimation, is that of making some assumptions concerning the demand evolution within a day (i.e. between different  $t$ ) so as to reduce the number of unknowns in a within day dynamic context. As an example, while the generation profile of each zone could be considered time varying among the different time slices, the distribution percentages between the different destination zones could be considered linked to territorial aspects more slowly varying across the day. On the basis of this theoretical considerations, the aim of the OD demand estimation in a within day dynamic context could be, more efficacely, that of estimating the generation profiles from each zone and for each  $t$  but, as distribution percentages, the average values in a time period  $T$  larger than  $t$ . In this way, the equation/unknown ratio can be pushed towards the desired "one" value and true generation profiles and average distribution percentages can be estimated with good quality independently on the quality of the a-priori OD estimates, as shown in Marzano et al. (2009).

#### 6.2.5 *Joint estimation of OD demand and travel behaviour parameters*

The demand simulator relies on estimates of OD demand, route choice model parameters and network travel times in order to accurately model the network and its demand patterns. Since calibration of OD demand flows and route choice separately leads to biased OD flows estimates, in the last years attention to methods for joint estimation of the demand parameters has grown. However, the literature on joint OD estimation and parameter calibration is still limited.

Liu and Fricker (1996) proposed a two-step heuristic method for sequential estimating OD flows and route choice parameters on uncongested networks. In the first step, the route choice parameters are fixed and OD flows are estimated by minimizing the difference between observed and modeled link flows. In the second step, the

link flows that were obtained from the first step are used to calibrate the route choice parameters using a maximum likelihood method. Iterations are repeated until the first derivative of the likelihood value approaches zero. However, apart from its ignorance of congestion effects, implementation of the model requires a complete set of link traffic counts.

Yang et al. (2001) proposed an optimization model for simultaneous estimation of OD flows and the travel cost coefficient for congested networks in a logit-based stochastic user equilibrium model. The model was formulated in the form of a standard differentiable, nonlinear optimization problem with analytical stochastic user equilibrium constraints. Explicit expressions of the derivatives of the stochastic user equilibrium constraints with respect to origin-destination demand, link flow, and travel-cost coefficient were derived and computed efficiently through a stochastic network-loading approach. A successive quadratic programming algorithm using the derivative information was applied to solve the simultaneous estimation model. This work, however, performed only static OD estimates. Also, due to the inherent non-convex property of the problem, the technique used might lead to local optima.

Balakrishna (2002) presented an iterative framework for joint calibration of dynamic OD flows and route choice models using several days of traffic sensor data. The methodology for the calibration of the OD estimation module was based on an existing framework adapted to suit the sensor data usually collected from traffic networks. A static OD matrix for the AM peak was adjusted systematically using a sequential GLS estimator to obtain dynamic OD matrices for the entire AM peak. The calibrated parameters included route choice model parameters, time-varying OD flows, variance-covariance matrices associated with measurement errors and a set of autoregressive matrices, which capture the special and temporal inter-dependence of OD flows.

Sundaram (2002) developed a simulation-based short-term transportation planning framework for joint estimation of dynamic OD flows and network equilibrium travel times. While the coefficients of the route choice model are not estimated, a consistent set of OD flows and travel times are obtained by iterating between an OD estimation module and a day-to-day travel time updating model. Sundaram's approach operates in two steps. Travel times are established for a given set of dynamic OD demands. The resulting equilibrium travel time estimates are used to re-calculate assignment matrices for OD estimation. Travel times may then be computed again based on the new OD estimates if convergence has not been attained.

He et al. (2001) proposed a MLE approach to estimate the parameters of dynamic OD demand and route choice simultaneously with consideration of dependencies among paths and links, as well as some flow propagation information by utilizing time-dependent traffic data and any historical traffic information. Consistent estimates of dynamic OD flows and the route choice probabilities were obtained using approximation of joint probability distribution functions of link traffic flows on a network. The proposed method can be applied using link flow data alone. It is also possible to estimate parameters with measurement errors and incomplete data, especially when traffic counts are available only on a few links in the network.

A further generalization is contained in Lo and Chan (2003), who proposed a procedure for the simultaneous estimation of the O-D matrix and route choice dispersion parameter for congested networks.

#### 6.2.6 *Disaggregate demand estimation from aggregate data*

The traditional approach of generating (prior) OD matrices through the four-step process is increasingly replaced by the use of activity-based travel demand models (Ortuzar and Willumsen, 2004; Bowman and Ben-Akiva, 1998). This development has been enabled by methodological progress on the behavioural modelling side, the increased availability of disaggregate data, and vast improvements in computational facilities.

A shortcoming of the current situation is that despite of the *disaggregate* nature of activity-based travel demand models, dynamic traffic simulations are still based on *aggregate* OD matrices. Consequently, any OD matrix estimator is constrained to the adjustment of trip-making behaviour, without access to the much richer behavioural information provided by a disaggregate activity-based travel demand model.

It is well-known by now that OD matrices can be avoided by coupling the activity-based travel demand model directly to the traffic simulation (Nagel and Flötteröd, 2009). In this setting, it is still possible to use traffic counts for demand estimation, only that now the disaggregate travel behaviour in the activity-based model is adjusted from the counts directly and without intermediate aggregation into an OD matrix. This methodology is based on a Bayesian argument, which is outlined in the following (Flötteröd et al., 2010).

A stochastic traffic assignment simulation generates realizations of network conditions and travel behaviour that can be considered as draws from a *prior distribution* that reflects the analyst's knowledge about the modeled system. Traffic counts constitute additional measurements that can be linked through a likelihood function to the disaggregate travel behaviour in the simulation (Flötteröd and Bierlaire, 2009). In a Bayesian framework, the demand estimation problem then becomes to adjust the activity based model such that it generates realizations of the posterior distribution of the travel behaviour given the measurements. The joint estimation of travel behaviour and demand model parameters is possible along the same lines.

This approach (i) accounts for and exploits the complex behavioural constraints implemented in the activity-based model, (ii) is applicable under the same technical premises as any OD matrix estimator, (iii) is capable of estimating OD matrices as a special case of an activity based model with very limited degrees of freedom, and (iv) has vast computational advantages over usual OD matrix estimators in that it constitutes a one-step estimator that adjusts the travel demand within the iterative loop of the simulation. This results in an almost negligible computational overhead when compared a plain simulation.

Flötteröd et al. (2009) demonstrate the applicability of this method to a real-world scenario with ten thousands of network links and hundreds of thousands of simulated travellers. The approach is implemented in a freely available software tool that reflects its continuous development (Flötteröd, 2009; 2010).

#### 6.2.7 Selection of link counts location

A theoretical issue strictly linked to the problem of o-d matrix correction using traffic counts is the proposition of methods for the optimal location of link count sections, that is the identification of the set of link flows providing for the maximum information with respect to the knowledge of the underlying o-d matrix, given a budget constraint. The problem is normally formalized in the literature through the definition of a reliability measure of the o-d matrix estimation based on traffic counts, and through the consistent proposition of optimization techniques or heuristics for finding a set of link counts (under given constraints) that maximizes this reliability measure. Yang et al. (1991) introduce the Maximum Possible Relative Error (MPRE) as reliability measure, and formulate a quadratic optimization problem under the constraints given by the equations expressing counted flows as a function of o-d matrix entries, in the hypothesis of error-free counts and assignment matrix. The same hypotheses are shared by Bierlaire (2002), who proposes theoretical measures related to the volume of the feasibility set of link flows and, due to the numerical difficulty in calculating the volume of a polytope in large dimensions, practically the Total Demand Scale (TDS) measure, defined as the difference between the maximal and the minimal total value of the demand (i.e. sum of the o-d matrix entries) consistent with counted flows, calculable by means of two constrained linear programming problems. Both Yang et al. (1991) and Bierlaire (2002) discuss the need for setting finite valued reliability measures, a circumstance addressed by Yang and Zhou (1998) in their work by means of the constraints defined by the so called o-d covering rule. In addition, Yang and Zhou (1998) propose other three rules for the optimal link count sections location, i.e. the maximal flow fraction, the maximal flow interception and the link independence rules. Notably, the issue of defining upper bounds for the feasibility set of the link flows is just a theoretical speculation, since from a practical standpoint proper limits can be defined, based on socio-economic characteristics of each traffic zone, e.g. number of generated trips not larger than the population in the morning peak hour. Furthermore, the quality of the method proposed by Yang and Zhou (1998) is strictly conditioned by the reliability of the required prior o-d matrix estimate.

A number of other methods have been also proposed in the literature to date, for instance Ehlert et al. (2006) generalize the work by Yang et al. (1991), introducing section-specific counting costs, and also count section weights into the optimization functions related to their information content (or entropy). Another research path deals with the identification of the whole set of link flows starting from an observed (i.e. counted) subset, as proposed by Hu et al. (2009) who operate with the sole topology of the network, requiring explicit path enumeration and without inferring about the underlying o-d matrix. Similarly, heuristics aimed at providing for a geographical/topological disaggregation of link flows are discussed by Yang et al. (2006). Furthermore, other network based flow measures can be taken into account as well, for instance plate scanning surveys which can be used, as proposed by Minguez et al. (2010), leading to the proposition of a mixed integer programming problem. Simonelli et al. (2011) propose an innovative methodology for addressing the issue of the optimal link count sections location, based on the proposition of a reliability measure in which the prior accuracy of the o-d matrix estimate, that is its statistical distribution rather its prior punctual estimate, is explicitly considered, together with its posterior distribution conditioned on a given subset of link count locations.

### 6.3 Supply models

In comparison with the demand parameters, the supply parameters calibration seems to have received relatively less attention, though it plays a critical role in determining network performance. Supply models mimic traffic dynamics using speed-density relationships and traffic phenomena of queue formation, dissipation and spillback. The number and nature of supply variables may vary depending on the level of detail employed while capturing traffic dynamics and queuing phenomena. Recently developed algorithms applied to macroscopic, mesoscopic and microscopic supply model calibration are reviewed in this section.

#### 6.3.1 *Macroscopic and mesoscopic supply calibration*

Macroscopic traffic models capture traffic dynamics through aggregate relationships derived by approximating vehicular flow as a fluid. Several macroscopic models are reported in the literature, including METANET (Messmer and Papageorgiou, 2001), EMME/2 (INRO, 2006), VISUM (PTV, 2006), SATURN (Van Vliet, 2009) and the cell transmission model (CTM, Daganzo, 1994).

Mesoscopic models are syntheses of microscopic and macroscopic modelling concepts. They couple the detailed behaviour of individual drivers' route choice behaviours with more macroscopic models of traffic dynamics. Examples of such systems include DynaMIT (Ben-Akiva et al., 2001, 2002) and DYNASMART (Mahmassani, 2002).

In both mesoscopic and macroscopic traffic simulation models, speed-density functions are critical for modelling traffic dynamics. Recent studies have employed systematic algorithms for the supply models calibration, in particular speed-density relationships, with varying degrees of success. The typical data used for the calibration of these parameters are sensor records of at least two of three primary traffic descriptors: speeds, flows and densities. The applications are classified as off-line (archived sensor data) and on-line (real-time sensor data).

##### 6.3.1.1 *Off-line calibration approaches*

Van Aerde and Rakha (1995) described the calibration of speed-flow profiles by fitting data from loop detectors on I-4 near Orlando, Florida. Network links were grouped based on the traffic characteristics observed at sensor locations. Speed-flow profile estimated for a link equipped with a sensor was allotted among links in a group. Similar approaches have been widely applied on networks of realistic size and structure.

Leclercq (2005) estimated four parameters of a two-part flow-density function with data from arterial segments in Toulouse, France. The function was comprised of a parabolic free-flow part and a linear congested regime. An interior point, conjugate gradient method was employed to optimize the fit to observed sensor flows, with the fitted flows obtained from the assumed flow-density function. A major drawback of this approach is local fitting. The estimated link performance functions reflect spot measurements at discrete sensor stations, and do not necessarily correspond to overall link dynamics (especially in the presence of congestion). The estimation procedure also does not enforce consistency across contiguous links or segments, stressing the need for an expanded approach that considers larger sections of the network.

Munoz et al. (2004) described a calibration methodology for a modified cell transmission model (MCTM) for a freeway stretch in California. Free-flow speeds were obtained through least squares by fitting a speed-flow plot through each detector's data. Free flow speeds for cells without detectors were computed by interpolating between the available speed estimates. In the case of bad or missing sensor data, a default of 60 mph was assumed. For the purpose of capacity estimation, the freeway was divided into congested and free-flow sections by studying speed and density contours from detector data. Capacities in the free-flow cells were set to be slightly higher than the maximum flow observed at the nearest detector. Bottleneck capacities were estimated to match the observed mainline and ramp flows just upstream of the free-flow part of the bottleneck. Speed-flow functions were obtained through constrained least squares on sensor data from congested cells.

Yue and Yu (2000) calibrated the EMME/2 and QRS II (Horowitz, 2000) models for a small urban network in the U.S. While no systematic calibration approach was outlined, the authors adjusted the free-flow travel times and turning fractions to match detector count data. Such ad-hoc procedures are unlikely to perform satisfactorily when applied to large-scale models and networks.

Ngoduy and Hoogendoorn (2003) calibrated METANET parameters using the Nelder-Mead method (a gradient-free algorithm working directly with objective function evaluations) for calibrating a freeway section in the Netherlands. The calibrated terms included fundamental diagram parameters such as free-flow speed, minimum speeds and maximum density, and other coefficients that capture the effects of merging, weaving and lane drops.

Park et al. (2006) applied DynaMIT to a network in Hampton Roads, VA and estimate speed-density functions for segments. They adopted the procedure in Van Aerde and Rakha (1995) and concluded that the initial calibration results need adjustments to improve DynaMIT's overall ability to estimate and predict traffic conditions.

Kunde (2002) presented a calibration of the supply models within a mesoscopic simulation system. A three-stage approach to supply calibration was outlined, in increasing order of complexity (single segment, sub-network, entire network) and applied to a large-sized mixed network in Irvine, California using DynaMIT. The results in the network wide calibration showed that the simultaneous perturbation stochastic approximation (SPSA) algorithm provides results comparable to those of the Box-Complex algorithm by using a much lower number of function evaluations, thus requiring much less run time.

#### 6.3.1.2 *On-line calibration approaches*

Van Arem and Van der Vlist (1992) developed an on-line procedure for the estimation of current capacity at a motorway cross-section. The method was based on two assumptions. First, it was assumed that there exists a "current" fundamental diagram which depends on prevailing conditions. A method for establishing such fundamental diagrams based on on-line measurements of flow, occupancy and speed was presented. The second assumption was that the capacity can be estimated using this fundamental diagram and the notion of "maximum" occupancy. The capacity was estimated by substituting the current maximum occupancy into the current fundamental diagram.

Tavana and Mahmassani (2000) used transfer function methods (bivariate time series models) to estimate dynamic speed-density relations from typical detector data. The parameters were estimated using the past history of speed-density data. The method was based on time series analysis, using density as a leading indicator. The resulting model was a descriptive rather than behavioural model to estimate speed and subsequently to predict its value for future time intervals.

Huynh et al. (2002) extended the work of Tavana and Mahmassani (2000) by incorporating the transfer function model into a simulation-based Dynamic Traffic Assignment (DTA) framework, DYNASMART. Furthermore, the estimation of speeds using the transfer function model was implemented as an adaptive process to a small mixed network in San Antonio, Texas, where the model parameters were updated on-line based on the prevailing traffic conditions. A nonlinear least squares optimization procedure was also incorporated into the DTA system to enable the estimation of the transfer function model parameters on-line. Results from simulation-based experiments confirmed that the adaptive model outperforms the non-adaptive model. The scope of this study, however, was limited to updating speeds on a single link using synthetic data. Qin and Mahmassani (2004) addressed the shortcomings of the previous approach by evaluating the same model with actual sensor data.

Wang and Papageorgiou (2004) presented a general approach to the real-time estimation of the complete traffic state in freeway stretches. They used a stochastic macroscopic traffic flow model, and formulated it as a state-space model, which they solved using an Extended Kalman Filter. This formulation allows dynamic tracking of time-varying model parameters by including them as state variables to be estimated. A random walk was used as the transition equations for the model parameters. Wang et al. (2007) presented an extended application of this approach.

#### 6.3.2 *Microscopic supply calibration*

Microscopic models capture traffic dynamics through detailed representations of individual drivers and vehicular interactions. Popular commercial microscopic software packages include CORSIM (FHWA, 2005), PARAMICS (Smith et al., 1995), AIMSUN2 (Barcelo and Casas, 2002), MITSIMLab (Yang and Koutsopoulos, 1996; Yang et al., 2000), VISSIM (PTV, 2006), DRACULA (Liu et al., 1995) and Trans-Modeler (Caliper, 2006).

Microscopic calibration involves estimation of driving behaviour parameters such as acceleration, lane-changing and car-following. Microscopic data are complex both to obtain and to calibrate. The difficulty of this calibration problem is that the data usually available are aggregate measurements of traffic characteristics. Aggregate cali-

bration is based on the interaction among all the components of the simulation model. Therefore, it is impossible to identify the effect of individual models on traffic flow when using aggregate data. In general, the aggregate calibration of microscopic supply parameters is a part of joint demand-supply calibration. Furthermore, the calibration of microscopic models does not provide very general results, meaning that the resulting set of parameters is often difficult to be used in applications in different locations.

A number of papers have been published on the subject of microscopic calibration using aggregate data. Hourdakis et al. (2003) presented a three-stage general and systematic methodology for manually calibrating microscopic traffic simulators. They first sought to match observed traffic flows by calibrating global parameters, such as vehicle characteristics. Then, local link-specific parameters, such as speed limits, were calibrated to match observed speeds. An optional third calibration stage was suggested, where any measure chosen by the user can be compared. A quasi-Newton algorithm was used for the solution of the various subproblems.

A simplex optimization approach for microscopic calibration was proposed by Kim and Rilett (2003). The simulated and observed traffic counts from TRANSIMS and CORSIM traffic micro-simulation models were matched under two different demand matrices. The gradient-free downhill simplex algorithm was used by Brockfeld et al. (2005) to calibrate a small set of supply parameters in a wide range of microscopic and macroscopic traffic models. Hollander and Liu (2005) used a similar simplex approach for calibrating a small urban network in the U.K. with DRACULA, by minimizing an objective function that expresses differences between observed and simulated travel time distributions.

The use of Genetic Algorithm (GA) for the microscopic parameters calibration was illustrated in several works. Ma and Abdulhai (2002) developed a parametric optimization tool GENOSIM for micro-simulation calibration. In GENOSIM, GAs are used by a generic and independent calibration tool to interface with any microscopic traffic simulation model and the model calibration process is transformed into an automated, systematic, and robust process. Kim and Rilett (2004) conducted calibration process with GA by using both CORSIM and TRANSIMS, and showed the benefits of using a GA for automated calibration. Kim et al. (2005) proposed a nonparametric statistical technique for use in an automated microscopic calibration procedure. GA was used to find the best parameters. The proposed method was applied to a real network by using VISSIM.

However, these approaches used a few selected calibration parameters due to the complexity of the optimization surface (i.e., stochastic nature of microscopic simulation models and the total number of combinations when all the possible calibration parameters are considered). Park and Qi (2005) adopted a statistical experimental design approach to reduce the number of combinations and also considered feasibility of the initial ranges of calibration parameters. The authors showed that if initial ranges of calibration parameters do not contain optimal solution (i.e., field condition), the simulation model cannot be calibrated even the optimization method finds optimal solution within the search region. This approach was successfully tested at an isolated signalized intersection using VISSIM simulation model.

Hollander and Liu (2008) presented several guidelines after reviewing several key aspects of the model calibration procedures used in some of the most recent publications: scope of the calibration problem, the formulation and automation of the calibration process (especially in its decomposition into sub-processes), the measures of the goodness of fit, the number of repeated runs. They state that many calibration methodologies are not rigorous enough in terms of the number of repetitions of the model used throughout the calibration procedure and that most authors still use traffic microsimulation for estimating mean values of various traffic measures, despite the fact that the stochastic nature of microsimulation creates an excellent opportunity for examining their variation.

The most optimization techniques applied for the calibration of supply model parameters are limited to simple networks and small parameter sets. To ascertain their suitability for overall model calibration, tests on larger networks and variable sets should be performed.

#### **6.4 Joint demand-supply calibration**

The calibration of demand and supply components has generally been attempted through a sequential procedure: supply parameters are calibrated first, assuming that demand inputs are known, then demand parameters are estimated with fixed supply ones. In this case, complex interactions between demand and supply components are ignored leading to not optimal results. Instead, simultaneous estimation of demand-supply parameters captures these interactions, ensuring consistency between the estimated parameters. This section focuses on methods for joint demand-supply calibration.

Since the problem of joint calibration of demand and supply parameters has only recently been developed, literature on this topic is limited. Mahut et al. (2004) described the calibration of a mesoscopic traffic simulator. Their software combines a microscopic network loading model and a mesoscopic routing engine. Iterations between the two components are used to establish dynamic equilibrium travel times on the network. In this calibration approach, the different model components are treated independently. OD flows are estimated by matching turning movement counts at major intersections with their simulated values. Capacities are estimated based on saturation flow rates and empirically derived downstream intersection capacities. Triangular volume-delay functions approximated based on posted speed limits and estimated capacities may not accurately reflect ground conditions, since drivers on average travel at speeds higher than the speed limit under uncongested conditions. The gap acceptance and route choice parameters are adjusted manually to minimize the objective function. While valuable insights are provided to support these adjustments, they remain case-specific, and may not generally be transferable.

Gupta (2005) demonstrated the calibration of the mesoscopic dynamic traffic assignment (DTA) model DynaMIT by using separate methodologies to calibrate the demand and supply parameters. In this work, supply and demand estimations were performed sequentially using real sensor count and speed data. Although the application has many limitations (such as sequential OD estimation and local supply fitting), it contributes significantly through the development of an observability test that allows the modeler to ascertain if unique OD flows may be estimated from the given sensor configuration and coverage.

Several recent studies have focused on calibrating both demand and supply model inputs for microscopic simulation. Mahanti (2004) calibrated the demand and select supply parameters for the MITSIMLab microscopic simulator by formulating the overall optimization problem in a GLS framework. The approach divides the parameter set into two groups: the OD flows (which may be estimated efficiently using existing tools) and the remaining parameters (including a route choice coefficient, an acceleration/deceleration constant in the car-following model, and the mean and variance of the distribution of drivers' desired speeds relative to the speed limit). An iterative solution method is implemented, with the OD flows estimated using the classical GLS estimator, and the parameters estimated by Box-Complex iterations.

Toledo et al. (2004) formulated the problem of jointly calibrating the OD flows, travel behaviour and driving behaviour components of microscopic models, using aggregate sensor data sources. The OD estimation step (utilizing a GLS formulation) was introduced as an explicit constraint, and a bi-level heuristic solution algorithm was used to solve for the three components iteratively. The use of the Box-Complex algorithm was reported for the estimation of select behavioural parameters.

Dowling et al. (2004) compared different simulated and observed measurements in separated stages of the calibration process: capacities to calibrate driving behaviour parameters, flows to calibrate route choice parameters, and finally travel times and queue lengths to fine-tune all parameters. The root mean squared error was used as goodness-of-fit in the comparison between simulation outputs and observed measurements in a small number of locations of a mixed network.

Jha et al. (2004) calibrated MITSIMLab for a large-scale network. They estimated driving behaviour parameters independently on a single freeway section for which OD flows could be inferred easily from sensor counts. Subsequently, OD flows, a route choice parameter and habitual travel times were obtained by iteratively calibrating each component individually until convergence. The authors discussed several practical issues relevant to large-scale model calibration, the most important being the effect of stochasticity and extremely low OD flows on the quality of the simulated assignment matrices used for the GLS-based OD estimation.

Balakrishna et al. (2003) developed an off-line DTA model calibration methodology for simultaneous demand and supply parameter estimation. Two algorithms were used for optimization: SPSA (Simultaneous Perturbation Stochastic Approximation) and SNOBFIT. It has been concluded that the two algorithms result in comparable parameter estimates, although SPSA does so at a fraction of the computational requirements.

Balakrishna (2006) and Balakrishna et al. (2007a) presented a calibration framework for simultaneous calibration of all supply and demand parameters typical to DTA models (e.g. OD flows, route choice parameters, capacities, speed-density parameters) using any available data (e.g. counts, speeds, densities, queue lengths). The problem was solved with SPSA algorithm. In the work proposed by Balakrishna et al. (2007b) the methodology was adapted for the simultaneous demand-supply calibration within a large-scale traffic micro-simulation model.

Antoniou et al. (2007) formulated the problem of on-line calibration of a DTA model as a nonlinear state-space model that allows the simultaneous calibration of all model parameters and inputs. The methodology is generic and flexible based on nonlinear extensions of Kalman filter method: the extended Kalman filter (EKF), the limiting EKF (LimEKF) and the unscented Kalman filter. The solution algorithms were applied to the on-line calibration of the state-of-the-art DynaMIT DTA model, and their use was demonstrated in a freeway network. The LimEKF showed accuracy that is comparable to that of the best algorithm but with vastly superior computational performance.

Vaze et al. (2009) presented a methodology for the joint calibration of demand and supply parameters of DTA model using multiple data sources (traffic count and AVI data). The calibration problem was formulated as a stochastic optimization problem, which was solved by using two algorithms, GA and SPSSA. The results indicated that use of AVI data significantly improves calibration accuracy.

## 6.5 Summary

Modern traffic simulation models replicate various traffic phenomena using interactions between complex demand and supply components. To provide realistic abstractions of network processes, a large set of model inputs and parameters should be calibrated before the model is applied. The goal of simulation model calibration is to obtain accurate depictions of the following traffic patterns:

- Dynamic OD demand matrices and route choice models to capture demand-side effects;
- Detailed representation of network capacities and traffic dynamics to replicate network supply phenomena;
- Complex interactions between travel demand and network supply. Simulation models should employ algorithms that are able to capture these interactions and accurately estimate queues, spillbacks and delays.

The presented literature review indicates several shortcomings in the state-of-the-art of the simulation model calibration using aggregate data. Primary, the most approaches calibrate demand and supply components independently, ignoring the effect of their interactions: supply parameters are calibrated first; then, demand parameters are estimated with fixed supply. Prevalent practices rely on heuristics and manual parameter adjustment approaches that are largely based on judgment. Moreover, these studies typically estimate a small subset of parameters deemed important in explaining observed data for specific networks and datasets, and typically do not perform sufficient iterations to ensure a high degree of accuracy.

In most of the existing methods for demand calibration, OD demand flows and route choice model parameters are estimated sequentially. OD demand is sometimes estimated ignoring congestion effects on the route choice and travel times; or the fixed point problem of OD estimation is solved using bi-level approach with a fixed assignment matrix in each step of the optimization procedure.

In regard to supply parameters calibration, capacities are generally approximated from sensor flow data and from recommendations in the Highway Capacity Manual. Speed-density functions are identified locally by fitting to sensor data.

Most studies in this area do not take into account the affect of various sources of variability in traffic counts and OD demand, such as variability in network conditions, events and incidents, weather, seasonality and so on.

The review of the existing methods for aggregate calibration can be concluded with the following main findings:

- Simultaneous calibration of all demand and supply parameters provides the most efficient estimates of the simulation model's inputs;
- The most common technique for traffic data collection is using loop detectors for traffic counts. Nevertheless, even though this tool can give time dependant information about utilization at specific places, in most of the applications, this type of data (at various places) does not produce sufficiently accurate information of the utilization of the network by vehicles. Using combined data from several tools leads to improving calibration accuracy.
- In comparison with manual search techniques, application of automated optimization algorithms (e.g. simplex algorithm) for calibration of simulation models is more efficient and less expensive. When the

simulator is stochastic, the calibration problem must also account for the inherent noise in model outputs.

- Estimation of both travel times and OD demand flows in one process or estimation of OD flows using travel time information as additional measurements provides significant improvement of estimation precision. Inclusion of travel time information into the calibration of the OD matrix makes the simulation model replicate the observed traffic conditions better.
- Simultaneous estimation of time-dependent OD demand flows across multiple time intervals helps capture the effect of long trips. The traditional sequential method ignores the contributions of OD departure flows to measurements in future time intervals.
- Calibration of supply parameters is significantly improved by capturing the network-level spatial and temporal correlations between the various measurements.

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