Pseudo-Measurements as Aiding to INS during GPS Outages

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Abstract

The complementary nature of INS and GPS navigation systems can be used advantageously in navigation systems design, as long as GPS measurements are available. However, when GPS measurements become unavailable (e.g., in urban environments or jamming), the INS navigation solution will drift with time due to inherent bias. We propose fusing pseudo-measurements with INS as a means to circumvent this problem. This is carried out using knowledge of the platform behavior, and translation of the operating environment's features into pseudo-measurement aiding for the estimation process. Field experiments show significant improvement in the accuracy of the estimated vehicle states.

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1. Introduction

The complementary nature of INS and GPS systems can be used advantageously in navigation systems design. Numerous methods have been proposed for the integration of GPS and INS information to provide a robust navigation solution (e.g., [1], [2]). As long as GPS measurements are available, the combined navigation solution is likely to provide satisfactory results. However, in several scenarios, e.g., tunnel crossing, navigation in roofed parking lots, or jamming, GPS measurements may become unavailable. In these cases, neither loosely nor tightly coupled approaches may be employed, thereby leaving the navigation solution to rely only on the INS. However, the INS navigation solution drifts over time due to its inherent bias. For a variety of applications, e.g., emergency services or military applications it is necessary to obtain a continuous and accurate navigation solution. Thus, usage of the standalone INS is insufficient. One approach to circumvent this problem is to fuse the INS data with another sensor (e.g., odometer or cameras). Another approach is to use knowledge of the platform's behavior and its operating environment to aid the INS, instead, or in addition to using actual measurements.

Knowledge of the platform's behavior may be employed by considering the physics of the problem at hand (platform dynamics and its operating environment) and translating it into a pseudo-measurement form. This concept was first proposed for target tracking by Tahk and Speyer [3]. Later Koifman and Bar-Itzhack [4] proposed aiding INS with aircraft dynamics equations. In ground navigation, Dissanayake et al. [5] incorporated the fact that vehicles do not, normally, slip or jump off the ground. Speed encoder data, coupled with vehicle's velocity pseudo-measurements, were then used to form a full-measurement velocity vector. Recently, Shin [6] and Godha [7] demonstrated the use of a velocity pseudo-measurement as aiding to a linear INS error model by perturbing the velocity governing equation.

This paper addresses a scenario in which a vehicle is equipped with a GPS receiver and an INS. At a certain point while it is traveling GPS measurements become unavailable (complete GPS denial) for reasons as those noted above. Our aim is to utilize knowledge of the vehicle dynamics characteristics and of the physical conditions in which it operates in order to aid the INS measurements. This is carried out by translating these conditions and characteristics into pseudo-measurements. The discussion is focused on short aiding periods, though pseudo-measurements may be useful for longer periods as well.

To demonstrate the application of the proposed model and verify the benefits of implementing pseudo-measurements for mitigating the INS drift, field tests are conducted using a low-cost MEMS INS. The MEMS INS is chosen because of its low-cost, small dimensions and low weight, which make it attractive for commercial use. To evaluate the contribution of the pseudo-measurements, fusion of pseudo-measurements with the MEMS INS is examined first, leaving aside integration with additional sensors. Fusion of other sensor measurements, with both pseudo-measurements and INS data, is then evaluated to examine the additional benefits of that information to the estimation process and the performance of the navigation solution.

The rest of the paper is organized as follows: Section 2 describes the coordinate frames and the INS error equations that will be utilized with the pseudo-measurements aiding. Section 3 presents the Kalman filter estimators, which we implement for the INS aiding. Section 4 presents a set of pseudo-measurement types, which are classified into two groups: one that utilizes the vehicle's dynamics, and the other that utilizes the operating environment. Section 5 presents results of case studies that demonstrate the impact of the pseudo-measurements on the navigation solution accuracy, and the integration of additional sensors into the estimation. Section 6 presents the conclusions.

2. INS Error Equations

The following coordinate frames are used in the presentation: Inertial frame (i-frame), Earth Centered Earth Fixed (e-frame) frame, North-East-Down (NED) frame (n-frame) and Body frame (b-frame). The i-frame origin is at the Earth center. The *x*-axis points towards the mean Vernal equinox, the *z*-axis is parallel to the Earth spin axis and the *y*-axis completes a right-handed orthogonal frame. The e-frame has its origin at the Earth center and rotates with the Earth spin. The *x*-axis points towards the Greenwich meridian, the *z*-axis is parallel to the Earth spin-axis and the *y*-axis completes a right-handed orthogonal frame. The n-frame has its origin fixed at the earth surface at the initial latitude/longitude position of the vehicle. The *x*-axis points towards the geodetic north, the *z*-axis is on the local vertical pointing down, and the *y*-axis completes a right-handed orthogonal frame. The b-frame origin is at the vehicle's center of mass. The *x*-axis is parallel to the vehicle longitudinal axis of symmetry, pointing forward, the *z*-axis points down and the y-axis completes a right-handed orthogonal frame.

Raw measurements from accelerometers and gyros are taken along the b-frame. They are transformed to the n-frame, where data integration is performed. The position in the n-frame is expressed by curvilinear coordinates $r^n = \begin{bmatrix} \phi & \lambda & h \end{bmatrix}^T$ where, ϕ is the latitude, λ is the longitude and h is the height above the Earth surface. Motion equations in the n-frame are given by [2]:

$$\begin{bmatrix} \dot{r}^n \\ \dot{v}^n \\ \dot{T}^{b\to n} \end{bmatrix} = \begin{bmatrix} D^{-1}v^n \\ T^{b\to n} f^b + g_1^n - \left(2\omega_{ie}^n + \omega_{en}^n\right) \times v^n \\ T^{b\to n}\Omega_{nb}^b \end{bmatrix} \quad D^{-1} = \begin{bmatrix} \frac{1}{(m+h)} & 0 & 0 \\ 0 & \frac{1}{(n+h)\cos(\phi)} & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{1}$$

where $v^n = \begin{bmatrix} v_N & v_E & v_D \end{bmatrix}$ is the vehicle velocity. $T^{b \to n}$ and $T^{n \to b}$ are the transformation matrices from the b-frame to the n-frame and vice-versa, respectively. f^b is the measured specific force. ω_{ie}^n is the Earth turn rate expressed in the n-frame. ω_{en}^n is the turn rate of the n-frame with respect to the Earth. g_1^n is the local gravity vector. *m* and *n* are the radii of curvature in the meridian and prime vertical respectively. Ω_{nb}^b is the skew-symmetric form of the body rate with respect to the n-frame given by:

$$\omega_{nb}^b = \omega_{ib}^b - T^{n \to b} \left(\omega_{ie}^n + \omega_{en}^n \right) \tag{2}$$

The INS mechanization equations provide no information about errors in the system states (caused by measurement errors) as they process raw data from the Inertial Measurement Unit (IMU) to estimate navigation parameters. The IMU outputs contain additional errors that cannot be compensated for. These errors are due to sensor uncertainties, including spurious magnetic fields and temperature gradients. To improve the INS performance it is necessary to develop an error model which describes how the IMU sensor errors propagate into navigation errors through the motion equation (Eq. (1)). These navigation errors are then corrected for in order to obtain a corrected navigation solution. Several models (e.g. [10], [13] and [2]) were developed to describe the time-dependent behavior of these errors. The classic approach is the perturbation analysis, in which navigation parameters are perturbed with respect to the true n-frame. Perturbation is implemented via a first-order Taylor series expansion of the states in Eq. (1). A complete derivation of this model can be found in [14], [6] and [15]. The state-space model is given by:

$$\begin{bmatrix} \delta\dot{r}^n \\ \delta\dot{v}^n \\ \dot{\varepsilon}^n \\ \delta\dot{b}_a \\ \delta\dot{b}_g \end{bmatrix} = \begin{bmatrix} F_{rr} & F_{rv} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ F_{rv} & F_{vv} & f^n & T^{b\to n} & 0_{3\times3} \\ F_{er} & F_{ev} & -\Omega_{in}^n & 0_{3\times3} & -T^{b\to n} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & \left(-\frac{1}{\tau_a}\right)_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & \left(-\frac{1}{\tau_g}\right)_{3\times3} \end{bmatrix} \begin{bmatrix} \delta r^n \\ \delta v^n \\ \varepsilon^n \\ \delta b_a \\ \delta b_g \end{bmatrix} + \begin{bmatrix} 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ T^{b\to n} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & -T^{b\to n} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & I_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & I_{3\times3} \end{bmatrix} \begin{bmatrix} \upsilon_a \\ \upsilon_g \\ \upsilon_{ba} \\ \upsilon_{bg} \end{bmatrix} \quad (3)$$

where the state vector consists of position error, velocity and attitude errors, and accelerometer and gyro bias/drift. A detailed description of the parameters in Eq. (3) is given in the Appendix.

3. Kalman Filter

We incorporate the INS dynamics with pseudo-measurements aiding in a Kalman filter. In general, a Kalman filter algorithm involves two steps: i) prediction of the state based on the system model, and ii) update of the state based on the measurements. The first step is prediction of the state and its associated covariance [1]:

$$\hat{x}_{k+1}^- = \Phi\hat{x}_k^+ ,\ \Phi = e^{F(t)\Delta t} \quad (4)$$

$$P_{k+1}^- = \Phi P_k^+ \Phi^T + Q_k \quad (5)$$

where the superscripts – and + represent the predicted and updated quantities (before and after the measurement update), respectively. x and P are the system state and the associated error covariance matrices, respectively. Φ is the state transition matrix from time *k* to time *k+1*. $F(t)$ is the system dynamics matrix. Q_k is the process-noise covariance-matrix [11] given by:

$$Q_k \approx \frac{1}{2}\left[\Phi_k G(t_k) Q(t_k) G^T(t_k) + G(t_k) Q(t_k) G^T(t_k) \Phi_k^T\right]\Delta t \quad (6)$$

where, $G(t)$ is the shaping matrix and Δt is the time step.

The second step is the measurement update:

$$K_{k+1} = P_{k+1}^- H_{k+1}^T \left(H_{k+1} P_{k+1}^- H_{k+1}^T + R_{k+1}\right)^{-1} \quad (7)$$

$$\hat{x}_{k+1}^{+} = \hat{x}_{k+1}^{-} + K_{k+1}\left(z_{k+1} - H_{k+1}\hat{x}_{k+1}^{-}\right) \quad (8)$$

$$P_{k+1}^{+} = \left(I - K_{k+1}H_{k+1}\right)P_{k+1}^{-} \quad (9)$$

where K_k is the Kalman gain. H_k is the measurement matrix. R_k is the measurement-noise covariance-matrix. z_k is the measurement.

4. Pseudo-Measurements

Pseudo-measurements take advantage of knowledge of the vehicle's dynamics and the physical conditions the vehicle experiences. This knowledge is utilized as measurements in the vehicle state-estimation process. As pseudo-measurements do not involve actual sensors, there is no cost associated with their introduction to the model. Furthermore, because they are continuously available, their update rate can be conveniently set to the INS operating sampling rate.

The introduced pseudo-measurements are classified into two categories: i) those expressing vehicle dynamics, and ii) those describing the vehicle operating environment. The separation into two categories is made because pseudo-measurements concerning vehicle's dynamics are unique to the platform type, whereas those concerning the operating environment may be applied regardless of the platform type (e.g., constant height pseudo-measurement can be applied to vehicles, vessels or aircrafts in the same manner).

4.1 Vehicle Dynamics

The dynamics of any platform is characterized by its acceleration and linear and angular velocities. As a result the states in the motion equation (Eq. (3)) contain six DOF dynamics. However, considering vehicle dynamics, the motion is limited along a given trajectory (a road). Thus, knowledge on the limitations posed by the vehicle dynamics can translated into pseudo measurements to reduce the DOF.

4.1.1 Body Velocity

As vehicles do not slide on the ground, velocities in the y_B and z_B directions can be assumed close to zero in the b-frame (namely $v_{B_y} \cong 0$ and $v_{B_z} \cong 0$; [5], [6]). Using this assumption, the computed velocity in the b-frame can be expressed as

$$v_B = \left(T^{b\to n}\right)^T v_N \tag{10}$$

After perturbing Eq. (10) and rearranging it, we obtain:

$$\delta v_B = T^{n\to b}\delta v_N - T^{n\to b}\left(v_N \times\right)\delta\varepsilon_N \tag{11}$$

where $\left(v_N \times\right)$ is the skew-symmetric form of the velocity vector. From the second and third rows of Eq. (11), the measurement equations are constructed:

$$z_k = \begin{bmatrix} v_{B_y} \\ v_{B_z} \end{bmatrix}_{INS} - \begin{bmatrix} \eta_{B_y} \\ \eta_{B_z} \end{bmatrix} \tag{12}$$

$$H_k = \begin{bmatrix} 0_{1\times3} & T_{12}^{b\to n} & T_{22}^{b\to n} & T_{32}^{b\to n} & v_E T_{32}^{b\to n} - v_D T_{22}^{b\to n} & v_D T_{12}^{b\to n} - v_N T_{32}^{b\to n} & v_N T_{22}^{b\to n} - v_E T_{12}^{b\to n} & 0_{1\times6} \\ 0_{1\times3} & T_{13}^{b\to n} & T_{23}^{b\to n} & T_{33}^{b\to n} & v_E T_{33}^{b\to n} - v_D T_{23}^{b\to n} & v_D T_{13}^{b\to n} - v_N T_{33}^{b\to n} & v_N T_{23}^{b\to n} - v_E T_{13}^{b\to n} & 0_{1\times6} \end{bmatrix} \tag{13}$$

where η_{B_y} and η_{B_z} are the measurements noise, inserted to compensate for a deviation from the zero velocity assumption. Eqs. (12) and (13) are used as input to the Kalman filter (Eqs. (7)-(9)).

4.1.2 Body Angular Velocity

A platform can have body angular velocities in all three directions: $\omega_{ib} = \begin{bmatrix} p & q & r \end{bmatrix}^T$. However, as vehicles travel on the ground, they only change their yaw (heading) angle (having $p = q = 0$), thereby leading to the following pseudo-measurements

$$z = \left[\omega_{ib}\right]_{INS} - \begin{bmatrix} 0 \\ 0 \\ r \end{bmatrix} \tag{14}$$

The first two rows of Eq. (14) can be used as pseudo-measurements. However, the body angular velocity is not modeled as a state in the system dynamics, only its bias (Eq. (3)). Body angular velocity can, therefore, be added either as a new state, or the state measurement in Eq. (14) can be converted to a pseudo-measurement on the body-angular velocity bias. In order to keep the state-space model simple, the second alternative is chosen. Namely, the assumption on the biases is that $\delta b_{xg} = \delta b_{yg} = 0$, as no angular velocity in these two directions should exist. The resulting measurement equations are given by:

$$z = \begin{bmatrix} \delta b_{xg\,INS} - \eta_{xg} \\ \delta b_{yg\,INS} - \eta_{yg} \end{bmatrix};\ H = \begin{bmatrix} 0_{1\times 9} & 1 & 0_{1\times 5} \\ 0_{1\times 10} & 1 & 0_{1\times 4} \end{bmatrix} \tag{15}$$

where η_{xg} and η_{yg} are the measurements noise, inserted to compensate for the inherent gyros drift.

4.1.3 Body Acceleration

Similar to the body velocity assumption, acceleration in the b-frame's y_B and z_B directions (a_{B_y}, a_{B_z}) is generally close to zero. As this assumption fails to hold for turning vehicles (where the centrifugal force contributes to body acceleration in the *y*-axis), only the *z*-axis component can be utilized as a pseudo-measurement, with the corresponding measurement given by:

$$z = [a_{zb}]_{INS} - 0 \tag{16}$$

Similar to the body velocity, the acceleration is not modeled as a state, only its bias. Therefore, the state measurement is translated into a bias measurement by assuming $\delta b_{za} = 0$. The equivalent measurement equations are given by:

$$z = \delta b_{za\,INS} - \eta_{za};\ \ H = \begin{bmatrix} 0_{1\times 14} & 1 \end{bmatrix} \tag{17}$$

where η_{za} is the measurement noise which is inserted to compensate for the inherent drift of the accelerometers.

4.2 Vehicle Operating Environment

Prior knowledge about the topography in which the vehicle is traveling (or about to travel), can be used to devise appropriate pseudo-measurements.

4.2.1 Constant Height

Usually, when driving in an urban environment, height may be assumed almost constant for short time intervals. Such pseudo-measurement was implemented [7] with respect to the e-frame by transforming the constant-height pseudo-measurement from a single position component in the n-frame to a full position vector measurement in the e-frame. However, with this approach, when the pseudo-measurement is incorrect, it affects the whole e-frame position vector. In contrast, pseudo-measurement errors in the n-frame do not affect the whole position vector. Assuming a constant height $h = h_c$, the measurement equations can be constructed as:

$$z = h_{INS} - \left(h_c + \eta_c\right) \quad H = \begin{bmatrix} 0 & 0 & 1 & 0_{1\times12} \end{bmatrix} \tag{18}$$

where η_c is the measurement noise, inserted to compensate for deviation from the constant height assumption.

4.2.2 Constant LLH Position

Similar to the constant height aiding, the vehicle's latitude and longitude can be assumed almost constant for short time intervals, in particularly when driving in urban environments. In order to obtain a full position measurement, the constant latitude, longitude and height assumptions are combined as follows:

$$z = \begin{bmatrix} \phi_{INS} - (\phi_c - \eta_\phi) \\ \lambda_{INS} - (\lambda_c - \eta_\lambda) \\ h_{INS} - (h_c - \eta_c) \end{bmatrix}; \quad H = \begin{bmatrix} I_3 & 0_{3\times12} \end{bmatrix} \tag{19}$$

where η_ϕ and η_λ are the measurements noise, inserted to compensate for deviations from the constant latitude and longitude assumptions.

This pseudo-measurement states that the vehicle is still (which is most likely not the case). However, for short time periods, the distance the vehicle travels cannot be too large and therefore, the measurement noise compensates for the vehicle motion.

4.2.3 Constant Slope

Another physical condition that may be used as a pseudo-measurement is that of a constant slope. When traveling on the road, the change in slope is usually moderate, that is $\dot{h}_k \simeq \dot{h}_0$. Following integration, this expression has the form $h_k = \dot{h}_0 t + h_0$, where h_0 is the initial height. Recall that the height rate of change, $\dot{h}$, is equal to the velocity component in the down direction, namely $\dot{h} = -v_D$, thus the constant slope implies $v_D = -\dot{h}_0$. The corresponding measurement equations are:

$$z = \begin{bmatrix} h_{INS} - (h_c + \Delta t \dot{h}_c + \eta_c) \\ v_{d\,INS} - (-\dot{h}_c + \eta_{\dot{c}}) \end{bmatrix}; \quad H = \begin{bmatrix} 0_{1\times2} & 1 & 0_{1\times12} \\ 0_{1\times5} & 1 & 0_{1\times9} \end{bmatrix} \tag{20}$$

where $\eta_{\dot{c}}$ is the measurement noise, inserted to compensate for deviations from the constant slope assumption. This pseudo-measurement is a generalization of the constant height pseudo-measurement and applies when height changes, e.g., when driving on a slope.

4.3 Combining pseudo-measurements

Finally, note that each pseudo-measurement can be used as a standalone aiding or can be combined with any other pseudo-measurement aiding. Since several pseudo-measurements work well in improving the position accuracy, and others work well in improving the velocity accuracy, it is only natural to combine them.

5. Field Experiments

To evaluate the contribution of the proposed pseudo-measurements, a field experiment was conducted in different urban environments. The vehicle was equipped with a Microbotics MIDG II [16] INS/GPS system. Noise densities of the acceleration and angular rate were $150\mu g/\sqrt{Hz}$ and $0.05\left(\deg/\sec\right)/\sqrt{Hz}$ respectively. Raw data from several trajectories with various vehicle dynamics and traffic conditions, including: varying topography, varying velocity and acceleration distributions, left/right turning, and roundabouts, was collected.

The combined GPS/INS solution (GPS measurements were available throughout the experiments.) was used as the nominal solution in this analysis. Raw data from the IMU sensors were combined with different pseudo-measurements offline, without using the GPS measurements for this analysis. For the computation, the 15-state filter, given in Eq. (3), was implemented.

The following error measures were utilized to evaluate the contribution of the various pseudo-measurements to the navigation solution:

$$\varepsilon_h(t) = h_{aiding}(t) - h_{nominal}(t) \tag{21}$$

$$\varepsilon_{ll}(t) = \sqrt{\varepsilon_{lat}^2(t) + \varepsilon_{long}^2(t)} \tag{22}$$

$$\varepsilon_{vel}(t) = \sqrt{\varepsilon_{vn}^2(t) + \varepsilon_{ve}^2(t) + \varepsilon_{vd}^2(t)} \tag{23}$$

where $\varepsilon_h(t)$ is the height error. $h_{nominal}(t)$ and $h_{aiding}(t)$ are the observed height and estimated height obtained with the pseudo-measurements aiding respectively. $\varepsilon_{ll}(t)$ and $\varepsilon_{vel}(t)$ are error measures of the latitude and longitude, and NED velocity components respectively. $\varepsilon_{lat}(t)$ and $\varepsilon_{long}(t)$ are the latitude and the longitude errors respectively. $\varepsilon_{vn}(t)$, $\varepsilon_{ve}(t)$ and $\varepsilon_{vd}(t)$ are the north, east and down velocity errors, respectively. These errors are defined in the same manner as in Eq. (21). Separate error measures for the height, latitude/longitude and velocity are adopted for the evaluation as different applications may find interest in different characteristics of the solution.

Sections 5.1-5.2 present results of fusing the INS and pseudo-measurements for two trajectories with different road characteristics. For each of the two trajectories all pseudo-measurements described in Section 4 were applied as aiding. Several combinations of these pseudo-measurements were also utilized, including: i) body-angular-velocity with body-velocity, ii) body-angular-velocity with constant slope, iii) body-velocity with constant LLH, and iv) constant LLH, body-velocity, body-angular-velocity, and body-acceleration.

Section 5.3 presents result of an additional trajectory that significantly violates the assumptions underlying the construction of the pseudo-measurements. In this experiment, as in the previous ones, the fusion of INS and the pseudo-measurements was evaluated. In addition, INS was fused with other vehicle sensors, including odometer and barometer. This experiment evaluates the contribution of the pseudo-measurements, the sensory inputs as independent aidings, and the fusion of the pseudo-measurements and the sensory inputs.

5.1 Trajectory I

In this trajectory, the stationary vehicle accelerated to $v = 60[km/h]$, and then kept velocity in the range between 60 and 80 [*km/h*]. The terrain in this experiment was relatively leveled, with height variations of ~5 [m] along the trajectory. The vehicle in this trajectory was driving in an urban area, making frequent turns and crossing several roundabouts, violating the body acceleration and body angular velocity pseudo-measurements assumptions.

Error measures of the INS mechanization solution are presented in Tables 1-3 for height, Lat/Long and velocity, respectively. These tables and the rest to follow present the INS-only solution and five types of pseudo-measurements aiding: body-velocity, which was previously proposed in [5], body-angular-velocity (Section 4.1.2), constant LLH (Section 4.2.2), body-acceleration (Section 4.1.3) and a combination of body velocity and constant LLH, termed combined PM. These pseudo-measurements provided the overall best performance compared to other aiding types. Body velocity was selected as a benchmark pseudo-measure as it was previously proposed in the literature. The pseudo-measurement providing the best result is highlighted in each table. The tables also show percent reduction in the error measures compared to the standalone INS solution.

Table 1 shows that drift resulting from an INS solution with no aiding, reaches a height error which is as high as $932[m]$ after $90[\sec]$. All pseudo-measurements, excluding the body angular velocity pseudo-measurement, improved the standalone INS solution by 70% to 99%. The combined PM aiding offered the best performance, lowering the height error to only $0.4[m]$ (eliminating more of 99% of the height error). The velocity error of the unaided INS (Table 2) was $55[m/s]$ after $90[\sec]$, while the body velocity and the constant LLH were less than $12[m/s]$ throughout the trajectory, yielding the best aiding result. In reference to the positioning error, Table 3 shows that the constant LLH pseudo-measurement obtained the best performance, lowering the Lat/Long error by a 72% after 90 sec.

Table 1: Trajectory 1: Height error [m] mean and percent reduction.

Aiding Type	**T=30[sec]**	**T=60[sec]**	**T=90[sec]**
INS only[m]	85	348	932
Body Velocity [m]	9 (89%)	88 (75%)	123 (86%)
Constant LLH [m]	0.3 (99.7%)	0.4 (99.8%)	0.4 (99.9%)
Body Angular Velocity [m]	85 (0%)	326 (6%)	863 (7%)
Body Acceleration [m]	10 (88%)	55 (83%)	287 (69%)
Combined PM [m]	0.2 (99.8%)	0.3 (99.8%)	0.4 (99.9%)

Table 2: Trajectory 1: Velocity error [m/s] mean and percent reduction.

Aiding Type	**T=30[sec]**	**T=60[sec]**	**T=90[sec]**
INS only[m/s]	15	46	55
Body Velocity [m/s]	5 (66%)	7 (85%)	4 (93%)
Constant LLH [m/s]	12 (20%)	12 (74%)	12 (78%)
Body Angular Velocity [m/s]	11 (27%)	41 (12%)	49 (11%)
Body Acceleration [m/s]	13 (13%)	39 (15%)	51 (8%)
Combined PM [m/s]	11 (27%)	11 (77%)	8 (85%)

The results show that the constant LLH pseudo-measurement obtained the best performance for the position components. Even though the body-velocity performance was higher in the velocity error, the constant LLH pseudo-measurement improved the INS standalone solution in all three categories. Referring to other pseudo measurements, the body acceleration pseudo-measurement improved both the height and Lat/Long error but failed improving the velocity error measure, the body-angular-velocity pseudo-measurement managed improving all error measures but only by 20% at best.

Table 3: Trajectory 1 Lat/Long error [m-rad] mean and percent reduction.

Aiding Type	**T=30[sec]**	**T=60[sec]**	**T=90[sec]**
INS only[m-rad]	0.025	0.17	0.43
Body Velocity [m-rad]	2.6 (Inc.)	17 (Inc.)	49 (Inc.)
Constant LLH [m-rad]	0.015 (40%)	0.07 (60%)	0.12 (72%)
Body Angular Velocity [m-rad]	0.022 (11%)	0.15 (11%)	0.36 (17%)
Body Acceleration [m-rad]	0.03 (21%)	0.17 (1%)	0.5 (15%)
Combined PM [m-rad]	0.041 (Inc.)	0.07 (58%)	0.11 (74%)

5.2 Trajectory II

As in the previous trajectory, the stationary vehicle accelerated first to $v = 60\left[km/h\right]$, and then maintained a velocity in the range between 60 and 80 [*km/h*]. The road in this experiment was unpaved, with many pits along it. Height variations along the road were ~25[m]. These road characteristics violate the assumptions of constant height, zero body-velocity and acceleration in the b-frame's y_B and z_B directions, and zero angular velocity. These violations have led to degraded performance of the related pseudo-measurements, relative to their application to the first trajectory. Nonetheless, most aidings improved the standalone INS solution. Results are listed in Tables 4-6. Despite the violation of some of the underlying assumptions, all pseudo-measurements improved the standalone INS solution for the height. The improvements were between 45%-99%, except for the body angular velocity pseudo-measurement. Results show that height change along the trajectory had little effect on the pseudo-measurements improvement relative to the standalone INS.

Table 4: Trajectory 2: Height error [m] mean and percent reduction.

Aiding Type	T=30[sec]	T=60[sec]	T=90[sec]
INS only[m]	201	689	1294
Body Velocity [m]	14 (92%)	108 (85%)	75 (94%)
Constant LLH [m]	1 (99%)	3 (99%)	1 (99%)
Body Angular Velocity [m]	195 (3%)	708 (Inc.)	1320 (Inc.)
Body Acceleration [m]	126 (37%)	388 (46%)	620 (52%)
Combined PM [m]	1 (99%)	2 (99%)	1 (99%)

The constant-LLH level of improvement suggests that the constant-height pseudo-measurement can perform well even if height along the trajectory varies. In this trajectory, the vehicle experienced small accelerations in it's y_B and z_B axes. As a result the velocity in these axes was no longer zero and thus violated the underlying body-velocity and acceleration pseudo-measurement assumptions.

Table 5: Trajectory 2: Velocity error [m/s] mean and percent reduction.

Aiding Type	T=30[sec]	T=60[sec]	T=90[sec]
INS only[m/s]	15	27	33
Body Velocity [m/s]	17 (Inc.)	30 (Inc.)	28 (16%)
Constant LLH [m/s]	15 (0%)	21 (23%)	21 (37%)
Body Angular Velocity [m/s]	16 (Inc.)	34 (Inc.)	35 (Inc.)
Body Acceleration [m/s]	13 (15%)	21 (24%)	19 (43%)
Combined PM [m/s]	15 (0%)	22 (20%)	22 (34%)

Table 6: Trajectory 2: Lat/Long error [m-rad] mean and percent reduction.

Aiding Type	T=30[sec]	T=60[sec]	T=90[sec]
INS only[m-rad]	0.02	0.11	0.20
Body Velocity [m-rad]	9 (Inc.)	12 (Inc.)	11 (Inc.)
Constant LLH [m-rad]	0.015 (27%)	0.11 (0%)	0.23 (Inc.)
Body Angular Velocity [m-rad]	0.015 (26%)	0.186 (Inc.)	0.51 (Inc.)
Body Acceleration [m-rad]	0.02 (0%)	0.100 (8%)	0.190 (6%)
Combined PM [m-rad]	0.019 (6%)	0.12 (Inc.)	0.27 (Inc.)

Table 5 shows that despite these violations, the body-acceleration pseudo-measurement was robust enough to improve the standalone INS velocity error by more than 15% throughout the trajectory. In contrast, the body-velocity pseudo-measurement caused the navigation error to increase. This result is related to the amount of measurement noise inserted into the estimation process. For the body-acceleration pseudo-measurement, the measurement noise was bigger than the acceleration applied by the vehicle due to the meandering road, and therefore had a smaller effect. For the body velocity pseudo-measurement, the resulting velocity was bigger than the measurement noise, leading to degraded results. The purpose of the noise added to the pseudo-measurements is to compensate for deviations from their underlying assumptions. In the experiments presented herein, constant noise level was given to all pseudo-measurements, thus explaining the differences. It is noted that if the topography of the road is known in advance, higher measurement noise can be used to circumvent variations in velocity caused by the meandering road. Concerning the Lat/Long errors, Table 6 shows that the body acceleration (throughout the trajectory) and the constant LLH (in the first 45 [sec]) pseudo-measurements improved the standalone INS results. The Lat/Long variation along this trajectory was similar to their variation in the trajectory I, however here the variation of the height was five times larger, causing the slight degradation in performance of the LLH pseudo-measurement

5.3 Trajectory III

The third trajectory was a steep winding road climbing a hill. In the first $60[\sec]$ segment, the change in height was about $30[m]$ while in the last $30[\sec]$ the change in height was $\sim 60[m]$. Overall, the climb is of ~100 [m]. In this trajectory the constant LLH and body velocity pseudo-measurements were incorporated by an odometer and a barometer to evaluate their influence on those sensors by aiding the INS. Odometers exist nowadays in most vehicles and thus may be used as an aiding to the INS. Nonetheless, [17] has shown that in some cases standalone INS obtains better results than the odometer, thus questioning the odometer actual contribution as a single aiding sensor to the INS. Barometers, on the other hand, are not commonly part of the vehicle but can be easily installed and used as an aiding sensor to the INS.

Table 7: Trajectory 3: Height error [m] mean and percent reduction.

Aiding Type	**T=30[sec]**	**T=60[sec]**	**T=90[sec]**
INS only[m]	146	490	1085
Body Velocity [m]	17 (88%)	56 (88%)	88 (92%)
Constant LLH [m]	1.8 (98%)	22 (95%)	60 (94%)
Odometer [m]	120 (18%)	201 (59%)	235 (80%)
Odometer + Barometer [m]	0.7 (99%)	0.8 (99%)	0.8 (99%)
Body Velocity + Constant LLH + Odometer [m]	2 (98%)	23 (95%)	60 (94%)
LL + Odometer + Barometer [m]	0.9 (99%)	0.6 (99%)	0.9 (99%)

Results of the error measures are listed in Tables 7-9. The odometer and barometer performances are listed there both as individual aidings and in a combined form. In addition, a fusion between the odometer, body velocity, and constant LLH pseudo-measurements was explored.

This choice of pseudo-measurements was based on the best performance obtained in the previous two sections. Another fusion scheme that was tested integrated the odometer, barometer and the constant LL pseudo-measurement (ignoring the constant height part since the height is obtained from the barometer). In terms of the pseudo-measurements acting as the only aiding data, results show that despite the change in height along the trajectory, the constant LLH still managed to considerably improve the INS solution. Regarding the sensory inputs, integration of the odometer and barometer yielded the best performance for both height and velocity error measure. Nonetheless, it greatly degraded the Lat/Long error measure (Table 9).

Table 8: Trajectory 3: Velocity error [m/s] mean and percent reduction.

Aiding Type	**T=30[sec]**	**T=60[sec]**	**T=90[sec]**
INS only[m/s]	11	19	28
Body Velocity [m/s]	15 (Inc.)	18 (5%)	17 (40%)
Constant LLH [m/s]	17 (Inc.)	17 (10%)	16 (42%)
Odometer [m/s]	25 (Inc.)	22 (Inc.)	24 (Inc.)
Odometer + Barometer [m/s]	1 (90%)	3 (84%)	6 (78%)
Body Velocity + Constant LLH + Odometer [m/s]	9 (18%)	10 (47%)	12 (57%)
LL + Odometer + Barometer [m/s]	17 (Inc.)	17 (10%)	15 (46%)

Using the odometer as the only aiding to the INS, improved only the height error (Table 7). Comparison of the odometer contribution to the constant LLH pseudo-measurement contribution shows that the latter performed better than the odometer in terms of height error reduction. It also improved the velocity and Lat/Long error measure, thus outperforming the odometer in all aspects. Results show that fusing the odometer with the constant LLH and body-velocity pseudo-measurements improved the odometer aiding in all three error measures. Therefore, the incorporation of pseudo-measurements is useful even with an odometer to enhance its performance.

Finally, fusion of both vehicle sensors and the Lat/Long pseudo-measurement significantly improved the Lat/Long error, yielding a 71% level of improvement after 90 [sec]. The derived results from all three tables show that the INS solution was improved by 70-90% after 90 [sec]. They are indicative to the contribution of pseudo-measurements in reducing INS errors, as well as their role within a fusion scheme that integrates additional sensory inputs.

Table 9: Trajectory 3 Lat/Long error [m-rad] mean and percent reduction.

Aiding Type	**T=30[sec]**	**T=60[sec]**	**T=90[sec]**
INS only[m-rad]	0.05	0.22	0.72
Body Velocity [m-rad]	19 (Inc.)	73 (Inc.)	81 (Inc.)
Constant LLH [m-rad]	0.05 (8%)	0.14 (37%)	0.21 (71%)
Odometer [m-rad]	155 (Inc.)	268 (Inc.)	437 (Inc.)
Odometer + Barometer [m-rad]	234 (Inc.)	379 (Inc.)	665 (Inc.)
Body Velocity + Constant LLH + Odometer [m-rad]	0.05 (8%)	0.138 (37%)	0.21 (71%)
LL + Odometer + Barometer [m-rad]	0.05 (8%)	0.139 (37%)	0.21 (71%)

5.4 Discussion

The three trajectories studied here feature different road/driving characteristics, including height variations of up to 100 [m], turns and crossing roundabouts as well as driving in an unpaved road. Nonetheless, results show that in most cases, introduction of the pseudo-measurements significantly reduces the navigation errors obtained with the standalone INS, in particular for time periods of up to 1 [min]. A general observation is that vehicle operating environment pseudo-measurements (Section 4.1) outperform those for vehicle dynamics (Section 4.2). This result may be attributed to the assumptions made on the vehicle dynamics, which are easily violated in real-world driving. The analysis shows that best performance for all three error measures examined (height, velocity, and latitude/longitude) is achieved using the constant LLH pseudo-measurement.

In addition, the experiments showed that incorporating pseudo-measurements with an odometer and a barometer helps improving the navigation performance. In particular, incorporating the body-velocity and constant LLH pseudo-measurements with the odometer greatly improved the standalone INS and the odometer and INS performances. Considering the availability of odometers in most vehicles, such aiding requires no overhead while attenuating the INS drift in all three aspects (height, velocity, and position determination).

6. Conclusions

This paper presented pseudo-measurements as aiding for MEMS INS over short periods of GPS outage (up to 90-seconds). The proposed pseudo-measurements were examined through various field experiments. These were designed so that the underlying assumptions of several of the pseudo-measurements were not exact in order to examine their robustness. Introduction of the pseudo-measurements reduced the navigation errors where best performance was obtained with constant LLH pseudo-measurement.

As the paper shows, pseudo-measurements can be employed for short time periods for aiding the INS in situations when GPS is not available and can be used as extra aiding with traditional in-vehicle sensors such as barometer or odometer to enhance the performance of the navigation system.

7. References

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8. Appendix

The following matrixes are associated with the INS equation of motion Eq. (2.3)

(A.1)

$$F_{rr} = \begin{bmatrix} 0 & 0 & \dfrac{-v_N}{(m+h)^2} \\ \dfrac{v_E \sin(\phi)}{(n+h)\cos^2(\phi)} & 0 & \dfrac{-v_E \sin(\phi)}{(n+h)^2 \cos^2(\phi)} \\ 0 & 0 & 0 \end{bmatrix} \quad F_{rv} = \begin{bmatrix} \dfrac{1}{(m+h)} & 0 & 0 \\ 0 & \dfrac{1}{(n+h)\cos(\phi)} & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

(A.2)

$$F_{vr} = \begin{bmatrix} -2v_E\omega_e \cos(\phi) - \dfrac{v_E^2}{(n+h)\cos^2(\phi)} & 0 & \dfrac{-v_N v_D}{(m+h)^2} + \dfrac{v_E^2 \tan(\phi)}{(n+h)^2} \\ 2\omega_e\left(v_N \cos(\phi) - v_D \sin(\phi)\right) + \dfrac{v_E v_N}{(n+h)\cos^2(\phi)} & 0 & \dfrac{-v_N v_D}{(n+h)^2} - \dfrac{v_E v_N \tan(\phi)}{(n+h)^2} \\ 2v_E\omega_e \sin(\phi) & 0 & \dfrac{v_E^2}{(n+h)} + \dfrac{v_N^2}{(m+h)} - \dfrac{2\gamma}{(R+h)} \end{bmatrix}$$

(A.3)

$$F_{vv} = \begin{bmatrix} \dfrac{v_D}{(m+h)} & -2\omega_e \sin(\phi) - \dfrac{2v_E \tan(\phi)}{(n+h)} & \dfrac{v_N}{(m+h)} \\ 2\omega_e \sin(\phi) + \dfrac{v_E \tan(\phi)}{(n+h)} & \dfrac{v_D + v_N \tan(\phi)}{(n+h)} & 2\omega_e \cos(\phi) + \dfrac{2v_E}{(n+h)} \\ -\dfrac{2v_N}{(m+h)} & -2\omega_e \cos(\phi) - \dfrac{2v_E}{(n+h)} & 0 \end{bmatrix}$$

(A.4)

$$F_{\varepsilon r} = \begin{bmatrix} -\omega_e \sin(\phi) & 0 & \dfrac{-v_E}{(n+h)^2} \\ 0 & 0 & \dfrac{v_N}{(m+h)^2} \\ -\omega_e \cos(\phi) - \dfrac{v_E}{(n+h)\cos^2(\phi)} & 0 & \dfrac{v_E \tan(\phi)}{(n+h)^2} \end{bmatrix} \quad F_{\varepsilon v} = \begin{bmatrix} 0 & \dfrac{1}{(n+h)} & 0 \\ -\dfrac{1}{(m+h)} & 0 & 0 \\ 0 & \dfrac{-\tan(\phi)}{(n+h)} & 0 \end{bmatrix}$$

(A.5)

$$F_{\varepsilon\varepsilon} = \begin{bmatrix} 0 & \omega_e \sin(\phi) + \dfrac{V_E \tan(\phi)}{(N+h)} & \dfrac{V_N}{(M+h)} \\ -\omega_e \sin(\phi) - \dfrac{V_E \tan(\phi)}{(N+h)} & 0 & -\omega_e \cos(\phi) - \dfrac{V_E}{(N+h)} \\ -\omega_e \cos(\phi) - \dfrac{V_E}{(N+h)\cos^2(\phi)} & \omega_e \cos(\phi) + \dfrac{V_E}{(N+h)} & 0 \end{bmatrix}$$

where $V^n \triangleq \begin{bmatrix} v_N & v_E & v_D \end{bmatrix}^T$ is the velocity vector in the n-frame and the rest of the parameters were defined in the text.