Vehicle Constraints Enhancement for Supporting INS Navigation in Urban Environments

Itzik Klein, Sagi Filin, Tomer Toledo

Faculty of Civil and Environmental Engineering

Technion – Israel Institute of Technology, Haifa 3200, Israel

Corresponding Author: iklein@technion.ac.il Phone: 972-4-8293080 Fax: 972-4-8295708

Published in Navigation 58(1), pp. 7-15, 2011

Abstract

The complementary nature of INS and GPS can be used advantageously in navigation systems design as long as GPS measurements are available. However, circumstances of complete GPS denial may occur in urban environments or in signal blockage scenarios. In such cases, the INS navigation solution drifts over time due to its inherent bias. To circumvent the INS drift, it is usually fused with external sensors. Nonetheless, use of external sensors increase the overall system cost. To reduce the need for external sensors, incorporation of vehicle constrains into the estimation process has been recently proposed. In order to improve the effect of these constraints, we propose implementing vehicle constrains both into the system and measurement models thereby enhancing the estimator performance. The paper demonstrates the contribution of the proposed approach via several field experiments. In all experiments, introducing system dynamics constraints reduced the navigation errors, compared to using only measurement constraints.

Keywords

MEMS INS, Vehicle Constraints, GPS Outages

INTRODUCTION

Fusion of GPS and INS is a common practice in modern navigation systems [1]. Such integration aims at utilizing the advantages of the both systems and compensating their weaknesses. In order to achieve this goal, several GPS/INS coupling architectures have been proposed [2], where some require a minimum of four GPS satellites (loosely coupling, [3]) while others apply when as few as one satellite is in view (tight coupling, [4]). However, in cases of GPS outage, which may occur in urban environments or in cases of signal blockage, the navigation solution is likely to rely on the INS standalone solution, which drifts in time, regardless of its grade.

To mitigate the navigation drift, fusion of the INS measurements with external sensors has been proposed using a variety of sensors, e.g., odometers [5], or magnetic sensors [6]. In general, the difference between the external sensor measurements and their INS counterparts is introduced into the INS error state filter as a means to estimate INS errors. Nonetheless, use of external sensors requires space and power and increases the cost of the overall system.

Incorporation of vehicle constraints into the estimation process has been recently proposed as a means to avoid the use of external sensors. These constraints translate a priori system knowledge into measurements, which are then incorporated into the estimator. This concept was first implemented for target tracking problems by Tahk and Speyer [7], and later Koifman and Bar-Itzhack [8] discussed aiding of INS with aircraft dynamics equations. In ground navigation, Brandt et al. [9] and Dissanayake et al. [10] utilized the fact that, normally, vehicles do not slip or jump off the ground as a pseudo-measurement of vehicle velocity, and recently, Shin [11], Godha [12], and Klein et al. [13] demonstrated the use a velocity pseudo-measurement as aiding to a linear INS error model by perturbing the velocity governing equation. Constraining the height was used by Lachapelle et al. [14] and by Godha and Cannon [12] for pedestrian and vehicular navigation. This constraint utilizes the fact that in urban environments vehicles usually maintain a constant height. Klein et al. [13] introduced ground vehicle dynamics related constraints. These are based on the fact that vehicles only accelerate forward or backward and only change their yaw angle (heading). In all of these cases, implementation of these vehicle constraints into the navigation estimation process was carried out by their translation into pseudo-measurements and taking the difference between them and their INS counterparts. The subtraction results were introduced into the INS error state filter, as "real" measurements from external sensors to estimate INS errors. In that process, the designer had to define in advance the amount of measurement noise that would compensate for possible discrepancies between the actual driving conditions and the underlying constraints related assumptions.

In this paper we propose means to enhance vehicle constraints performance in the estimation of the INS error states. We argue that since vehicle constraints represent the actual physical behavior of a vehicle on a surface, these constraints can be enforced regardless of the IMU measurements in the modeling of the system dynamics of the INS error state model. The vehicle constraints enforcement on the IMU measurements is safely applied since the focus here is on short time periods and on situations where a vehicle is travelling in an urban environment and experiences low-dynamics. As an example, consider the body velocity constraint that assumes zero velocity in the z-axis of vehicle body frame and thus zero acceleration in this direction. In the classical implementation, the zero velocity constraint is inserted only into the measurement model; yet in the proposed methodology, in addition to the measurement model, zero acceleration in the z-axis is inserted into the system model despite any different reading of the IMU in that direction. Such insertion contributes to an improved performance and a decrease of the INS navigation error. The proposed methodology is tested in several urban road experiments with a MEMS INS showing improvement of the classical vehicle constraints implantation. Whereas, cases of partial GPS availability may occur and be used in urban environments [15], total GPS denial is the case addressed here for all the experiments and the aim is to mitigate the INS drift until GPS becomes available once again. Additionally, as odometers are fused with the INS in many applications, we show that the proposed methodology improves performance compared to previous means to exploit vehicle constraints.

The rest of the paper is organized as follows: Section 2 provides background on INS error equations. Section 3 describes the application of vehicle constraints in the standard form and in the newly proposed form. Section 4 analyzes the proposed method via a set of experiments and presents some illustrative examples, and Section 5 provides conclusions of this research.

INS ERROR EQUATIONS

The navigation frame is defined as the one where the *x*-axis points towards the geodetic north, the *z*-axis is on the local vertical pointing down, and the *y*-axis completes a right-handed orthogonal frame. Position in the navigation frame is expressed by curvilinear coordinates $r^n = \begin{bmatrix} \phi & \lambda & h \end{bmatrix}^T$ where, ϕ is the latitude, λ is the longitude and h is the height above the Earth surface. Motion equations in the n-frame are given by [1]:

$$\begin{bmatrix} \dot{r}^n \\ \dot{v}^n \\ \dot{T}^{b\to n} \end{bmatrix} = \begin{bmatrix} D^{-1} v^n \\ T^{b\to n} f^b + g_1^n - \left(2\omega_{ie}^n + \omega_{en}^n\right) \times v^n \\ T^{b\to n} \Omega_{nb}^b \end{bmatrix} \tag{1}$$

$$D^{-1} = \begin{bmatrix} \frac{1}{(M+h)} & 0 & 0 \\ 0 & \frac{1}{(N+h)\cos(\phi)} & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad (2)$$

where $v^n = \begin{bmatrix} v_N & v_E & v_D \end{bmatrix}$ is the vehicle velocity; $T^{b \to n}$ and $T^{n \to b}$ are the transformation matrices from the body frame (The *x*-axis is parallel to the vehicle longitudinal axis of symmetry, pointing forward, the *z*-axis points down and the y-axis completes a right-handed orthogonal frame) to the n-frame and vice-versa, respectively; f^b is the measured specific force; ω_{ie}^n is the Earth turn rate expressed in the n-frame; ω_{en}^n is the turn rate of the n-frame with respect to the Earth; g_1^n is the local gravity vector, *M* and *N* are the radii of curvature in the meridian and prime vertical respectively; and Ω_{nb}^b is the skew-symmetric form of the body rate with respect to the n-frame given by:

$$\omega_{nb}^b = \omega_{ib}^b - T^{n \to b} \left(\omega_{ie}^n + \omega_{en}^n \right) \quad (3)$$

The INS mechanization equations provide no information about errors in the system states (caused by measurement errors) as they process raw data from the Inertial Measurement Unit (IMU) to estimate navigation parameters. Linking the IMU measurement errors and INS states is established through an error model which is derived using perturbations, implemented via a first-order Taylor series expansion of the states in Eq. (1). A complete derivation of this model can be found in Britting [16] and Shin [11]. The state-space model is given by:

$$\begin{bmatrix} \delta\dot{r}^n \\ \delta\dot{v}^n \\ \dot{\varepsilon}^n \\ \delta\dot{b}_a \\ \delta\dot{b}_g \end{bmatrix} = F \begin{bmatrix} \delta r^n \\ \delta v^n \\ \varepsilon^n \\ \delta b_a \\ \delta b_g \end{bmatrix} + G \begin{bmatrix} \upsilon_a \\ \upsilon_g \\ \upsilon_{ba} \\ \upsilon_{bg} \end{bmatrix} \tag{4}$$

$$F = \begin{bmatrix} F_{rr} & F_{rv} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ F_{rv} & F_{vv} & F^n & T^{b\to n} & 0_{3\times3} \\ F_{er} & F_{ev} & -\Omega_{in}^n & 0_{3\times3} & -T^{b\to n} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & \left(-1/\tau_a\right)_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & \left(-1/\tau_g\right)_{3\times3} \end{bmatrix}, \; G = \begin{bmatrix} 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ T^{b\to n} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & -T^{b\to n} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & I_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & I_{3\times3} \end{bmatrix} \tag{5}$$

where the state vector consists of position error, velocity and attitude errors, and accelerometer and gyro bias/drift. A detailed description of the parameters in Eq. (4) is given in the appendix.

We incorporate the INS error dynamics with pseudo-measurements aiding via a Kalman filter (see Appendix B). To that end, we define $x_{KF} = \left[\delta r^n \quad \delta v^n \quad \varepsilon^n \quad \delta b_a \quad \delta b_g\right]^T$ as the error state vector while the system dynamics and shaping matrices are defined in Eqs. (4)-(5).

VEHICLE CONSTRAINTS

Vehicle constraints, aka non-holomonic constraints, take advantage of knowledge of the vehicle's dynamics and the physical conditions the vehicle experiences. This knowledge is utilized as measurements in the vehicle state-estimation process. Three types of constraints are addressed in this presentation, including: i) body-velocity, ii) constant-height, and iii) body angular velocity (denoted VC, HC, and AVC, respectively). As these measurements are continuously available, their update rate is set to the INS operating sampling rate.

BODY VELOCITY CONSTRAINT

A body velocity constraint utilizes the fact that vehicles travel on the ground and do not slide. Therefore, velocities in the body frame along the y_B and z_B directions can be assumed to be almost zero [11], namely $v_{B_y} \cong 0$ and $v_{B_z} \cong 0$, and the computed velocity in the body frame can be expressed as

$$v^b = \left(T^{b\to n}\right)^T v^n \tag{6}$$

Perturbing Eq. (6) and rearranging it, leads to

$$\delta v^b = T^{n\to b}\delta v^n - T^{n\to b}\left(v^n \times\right)\delta\varepsilon^n \tag{7}$$

where $\left(v^n \times\right)$ is the skew symmetric form of the velocity vector. From the second and third rows of the vector in Eq. (7), the measurement equations have the form:

$$z_{VC} = \begin{bmatrix} v_{B_y,INS} - 0 \\ v_{B_z,INS} - 0 \end{bmatrix} + \begin{bmatrix} \eta_y \\ \eta_z \end{bmatrix} \tag{8}$$

$$H_{VC} = \begin{bmatrix} 0_{1\times3} & T_{12}^{b\to n} & T_{22}^{b\to n} & T_{32}^{b\to n} & v_E T_{32}^{b\to n} - v_D T_{22}^{b\to n} & v_D T_{12}^{b\to n} - v_N T_{32}^{b\to n} & v_N T_{22}^{b\to n} - v_E T_{12}^{b\to n} & 0_{1\times6} \\ 0_{1\times3} & T_{13}^{b\to n} & T_{23}^{b\to n} & T_{33}^{b\to n} & v_E T_{33}^{b\to n} - v_D T_{23}^{b\to n} & v_D T_{13}^{b\to n} - v_N T_{33}^{b\to n} & v_N T_{23}^{b\to n} - v_E T_{13}^{b\to n} & 0_{1\times6} \end{bmatrix} \tag{9}$$

where η_y and η_z are the measurement noise value for compensating possible discrepancies in the assumptions made on the zero body velocity. Eqs. (8) and (9) are used as inputs to the Kalman filter Eqs. (B.4)-(B.6).

For the added constraint, we begin by identifying the required changes in the system matrix F (Eq. (4)) when implementing the body velocity constraint. Assuming that $v_{B_y} \cong 0$ and $v_{B_z} \cong 0$, the corresponding body-accelerations, a_{B_y} and a_{B_z}, must also equal to zero

$$a_{B_y} = 0 \quad a_{B_z} = 0 \tag{10}$$

The measured body acceleration vector is expressed in its skew-symmetric matrix form, F^n, in the system matrix (Eq. (5)). Thus, F^n should be modified to include the body velocity constraint. Let, $\tilde{f}^b$ be the IMU acceleration measurement vector

$$\tilde{f}^b \triangleq \begin{bmatrix} \tilde{a}_{B_x} \\ \tilde{a}_{B_y} \\ \tilde{a}_{B_z} \end{bmatrix} = \begin{bmatrix} a_{B_x} + \upsilon_{acc-x} \\ a_{B_y} + \upsilon_{acc-y} \\ a_{B_z} + \upsilon_{acc-z} \end{bmatrix} \tag{11}$$

where, $\begin{bmatrix} a_{B_x} & a_{B_y} & a_{B_z} \end{bmatrix}$ is the actual vehicle acceleration vector in the b-frame and $\begin{bmatrix} \upsilon_{acc-x} & \upsilon_{acc-y} & \upsilon_{acc-z} \end{bmatrix}$ represents measurement noise. Introducing the body velocity constraint (Eq. (10)) into Eq. (11) yields

$$\tilde{f}^b = \begin{bmatrix} a_{B_x} + \upsilon_{acc-x} \\ \upsilon_{acc-y} \\ -g + \upsilon_{acc-z} \end{bmatrix} \tag{12}$$

Eq. (12) stands for the IMU readings when the body velocity constraint is valid during the measurement, yet it consists of IMU noise which under those circumstances should have been zero. In order to remove the IMU measurement noise in the *y* and *z* axes to fulfill the body velocity constraint, we replace the IMU measurements in those directions with

$$f^b = \begin{bmatrix} \tilde{a}_{B_x} \\ 0 \\ -g \end{bmatrix} \tag{13}$$

as the measured body acceleration. The corresponding accelerations in the n-frame, are given by

$$f_{VC-EVC}^n = T^{b\to n} f^b = \begin{bmatrix} T_{11}^{b\to n}\left(\tilde{a}_{B_x}\right) + T_{13}^{b\to n}\left(-g\right) \\ T_{21}^{b\to n}\left(\tilde{a}_{B_x}\right) + T_{23}^{b\to n}\left(-g\right) \\ T_{31}^{b\to n}\left(\tilde{a}_{B_x}\right) + T_{33}^{b\to n}\left(-g\right) \end{bmatrix} \tag{14}$$

Eq. (14), in its skew-symmetric from replaces the full-IMU readings sub-matrix F^n in the system matrix F (Eq. (5)). This way, the body velocity constraint is modeled inside the system matrix.

HEIGHT CONSTRAINT

The height constraint assumes that elevation remains almost constant for short time periods. Assuming $h = h_c$, and consequently $v_D = -\dot{h} = 0$, the measurement equations can be constructed as:

$$z_{HC} = \begin{bmatrix} h_{INS} - h_c \\ v_D - 0 \end{bmatrix} + \begin{bmatrix} \eta_h \\ \eta_{vd} \end{bmatrix};\ H_{HC} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0_{1\times 8} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0_{1\times 8} \end{bmatrix} \tag{15}$$

where η_h and η_{vd} are the measurement noise value for compensating possible discrepancies in the assumptions made on the constant height and zero velocity in the down direction.

Prior to describing the added height constraint, notice that $v_D = 0$, and thus $\dot{v}_D = 0$. In order to model the height constraint in the system matrix, we rewrite the second vector equation in Eq. (1)

$$\dot{v} = T^{b \to n} f^b + g_1^n - \left(2\omega_{ie}^n + \omega_{en}^n\right) \times v^n \tag{16}$$

Let

$$f^n = T^{b \to n} f^b \triangleq \left[\tilde{f}_N \quad \tilde{f}_E \quad \tilde{f}_D \right]^T \tag{17}$$

Focusing on short periods, the last term in Eq. (17) can be neglected (its value is also small in practice). The down-axis equation from Eq. (16) is thus reduced to

$$\dot{v}_D = f_D + g \tag{18}$$

Modeling the height constraint $\dot{v}_D = 0$ we have from Eq. (18) $f_D = -g$, thus the measured down-axis acceleration in the n-frame equals to the gravity magnitude in negative sign. Namely, the measured accelerations in the n-frame Eq. (17), which are required for the calculation of the system matrix F in Eq. (4), are given by

$$f_{HC-EVC}^n = \begin{bmatrix} \tilde{f}_N \\ \tilde{f}_E \\ -g \end{bmatrix} \tag{19}$$

ANGULAR VELOCITY CONSTRAINT

Finally, among the three body angular velocities $\omega_{ib} = \left[p \quad q \quad r \right]^T$, only change in the heading angle (having $p = q = 0$) is likely, as vehicles travel on the ground. Thus:

$$z = \left[\omega_{ib} \right]_{INS} - \begin{bmatrix} 0 \\ 0 \\ r \end{bmatrix} \tag{20}$$

The first two rows of Eq. (11) can be used as pseudo-measurements. However, the body angular velocity is not modeled as a state in the system dynamics, only its bias (Eq. (2)). Body angular velocity is, therefore, added as a state measurement in Eq. (20) when converted to a pseudo-measurement on the body-angular velocity bias. Namely, the assumption on the biases is that $\delta b_{xg} = \delta b_{yg} = 0$, as no angular velocity in these two directions should exist. The measurement equations are:

$$z_{AVC} = \begin{bmatrix} \delta b_{xg\,INS} - \upsilon_{xg} \\ \delta b_{yg\,INS} - \upsilon_{yg} \end{bmatrix} + \begin{bmatrix} \eta_{xg} \\ \eta_{yg} \end{bmatrix}; \; H_{AVC} = \begin{bmatrix} 0_{1\times 9} & 1 & 0_{1\times 5} \\ 0_{1\times 10} & 1 & 0_{1\times 4} \end{bmatrix} \tag{21}$$

where υ_{xg} and υ_{yg} are the expected known biases of the gyros which depend on their quality, and η_{xg} and η_{yg} are measurements noise, inserted to compensate for the inherent gyros drift.

Contrary to the previous two constraints, the angular velocity is neither part of the state-vector nor used directly in the estimator system dynamics model. On the other hand, the angular velocity product builds the transformation matrix from the b-frame to the n-frame, and is part of the dynamics model. Therefore, in the proposed approach, it can be altered to comprise the prior knowledge on the angular velocity. Since vehicles can only turn on the surface (ignoring irregular road conditions such as ice) they experience angular velocity only in the z-axis direction, and so reduce Eq. (3) to

$$\omega_{nb}^{b} = \begin{bmatrix} 0 & 0 & \omega_{ib-z}^{b} \end{bmatrix}^{T} - T^{n\to b}\left(\omega_{ie}^{n} + \omega_{en}^{n}\right) \tag{22}$$

Replacing Eq. (3) with Eq. (22) we from a modified skew-symmetric form of the body rate with respect to the n-frame Ω_{nb}^{b} which is used in Eq. (1) to yield a new transformation matrix $T^{n\to b}$. This transformation matrix is inserted into F and G in Eq. (4). In that manner, the angular body velocity constraint is modeled in the system and shape matrices through the transformation matrix.

Table 1 summarizes and presents the implemented constraints in the enhanced vehicle constraint (EVC) approach.

Table 1: Constraints applied to the EVC approach

Vehicle Constraint type	EVC approach
VC	$v_{B_y} = 0$, $v_{B_z} = 0$ & Eq. (14)
HC	$h = h_c$, $v_D = 0$ & $f_D = -g$,
AVC	$\delta b_{gx} = 0$, $\delta b_{gy} = 0$ & $p = 0$, $q = 0$

ANALYSIS & DISCUSSION

To demonstrate the contribution of the proposed EVC approach, experiments featuring data collected using a MEMS INS while driving in an urban environment are presented. The vehicle was equipped with a Microbotics MIDG II [17] INS/GPS system. Noise densities of the acceleration and angular rate were $150\mu g / \sqrt{Hz}$ and $0.05(\deg/\sec)/\sqrt{Hz}$ respectively. Raw data from five trajectories with various vehicle dynamics and traffic conditions, including: varying topography, varying velocity and acceleration distributions, left/right turning, and roundabouts, were collected. The duration of each trajectory was 90 seconds. Those trajectories and the vehicle dynamics while traveling on them are summarized in Table 2. All tests but #5 were carried out on paved roads and included left or right turns. The average height change along trajectory 1 was about 9 meters while in the rest of the trajectories it was less than 2 meters. Trajectories 1 and 5 where specifically chosen so that the underlying assumptions behind the vehicle constraints are violated. In a typical urban environment, trajectory 1 (featuring significant elevation change) and trajectory 5 (unpaved road) are not likely to be encountered but they are mentioned here to evaluate the EVC approach.

Table 2: Trajectories topography and vehicle dynamics

	Average Velocity [m/s]	Average Height Change [m]	Turning	Road Type
Trajectory 1	14	9	Left/right	Paved
Trajectory 2	9	0.5	Straight line with roundabouts	Paved
Trajectory 3	11	2	Left/right	Paved
Trajectory 4	15	2	U turn	Paved
Trajectory 5	12	0.5	Straight Line	Unpaved

The combined GPS/INS solution (GPS measurements were available throughout the experiments, and a GPS solution, contrary to a differential one) was used as the nominal solution in this analysis. Raw data from the IMU sensors were combined with both implementations of the vehicle constraints offline, without using the GPS measurements for this analysis. The 15 error-state filter, given in Eqs. (4)-(6), was implemented for the computation,.

To evaluate the contribution of the proposed approach, position and velocity error measures are examined. To that end, the following error measure is utilized:

$$\varepsilon_q(t) = q_{aiding}(t) - q_{nominal}(t) \tag{23}$$

where $\varepsilon_q(t)$ is the error for state q, $q_{aiding}(t)$ is the state history obtained from the aiding of vehicle constraints and $q_{nominal}(t)$ is the nominal state history. The position and velocity errors are obtained from

$$\varepsilon_{pos} = \sqrt{\left(\varepsilon_{lat}\right)^2 + \left(\varepsilon_{long}\right)^2 + \left(\varepsilon_h\right)^2} \tag{24}$$

$$\varepsilon_{vel}(t) = \sqrt{\varepsilon_{vn}^2(t) + \varepsilon_{ve}^2(t) + \varepsilon_{vd}^2(t)} \tag{25}$$

where $\varepsilon_h(t)$, $\varepsilon_{lat}(t)$ and $\varepsilon_{long}(t)$ are the height, latitude and the longitude errors respectively and $\varepsilon_{vn}(t)$, $\varepsilon_{ve}(t)$ and $\varepsilon_{vd}(t)$ are the north, east and down velocity errors, respectively. Once the position and velocity errors along a single trajectory were calculated, an average position and velocity value was obtained for each trajectory. The mean error, obtained from the five trajectories, is listed in the following tables that compare the standalone INS performance to the proposed methodology via several vehicle constraints as aiding.

EVC EVALUATION

Four types of vehicle constraints were applied as aiding to the INS, including the: i) VC, ii) VC+AVC, iii) VC+HC, and iv) VC+HC+AVC. The performance of the standalone HC and AVC vehicle constraints, as well as their combination (HC +AVC), was poorer and is therefore not detailed here. The reason for their limited performance lies in the observability of the system when fused with these constraints. When applying, e.g., the HC constraint, only the attitude and downward velocity component are affected, improving their estimation but having little influence on the other position and velocity components. As our interest is in improving the estimation of the complete position and velocity vectors, the overall HC contribution becomes limited. In contrast, the application of the VC directly contributes to both velocity and attitude states estimation, and therefore improves the position estimates. To further improve the VC performance, it is fused with the HC (improving altitude and downrange velocity) and the AVC (improving attitude), increasing the observability of the system and thus, theoretically, the estimation performance.

Evaluating the enhanced vehicle constraints for all the five trajectories, Table 3 lists the mean position error for the EVC approach for durations of 30, 60 and 90 seconds. In addition, improvement rate relative to the INS navigation solution is given for each case.

Table 3: Mean position error obtained from five trajectories [m]

Time	INS	VC	VC+AVC	VC+ HC	VC+ HC+AVC
30 [sec]	19	9.0 (48.1%)	8.9 (48.9%)	8.9 (48.3%)	9.2 (47.2%)
60 [sec]	133	79.7 (42.9%)	76.1 (45.1%)	82.7 (42.1%)	77.5 (43.4%)
90 [sec]	302	185.8 (40.6%)	180.6 (41.7%)	184.9 (43.1%)	175.7 (40.8%)

Table 3 shows that as time progresses, the EVC performance decreases. Yet in all cases, at least a 40% level of improvement is achieved relative to the standalone INS. Two observations can be drawn from Table 3: i) the standalone VC did not obtain the best performance regardless of the addressed time-period, and ii) the VC+AVC obtained best performance after 30 and 60 seconds, while after 90 seconds VC+HC constraint did. As expected from the observability analysis, the

VC is the major contributor to the improvement of the standalone INS. The HC managed improving the altitude channel by more than 70% in all examined trajectories (data not shown). Additionally, the AVC managed obtaining similar performance as the standalone VC in the velocity vector, yet failed to improve significantly the position vector.

Table 4 presents the mean velocity error for the EVC approach for 30, 60 and 90 seconds. In addition, the rate of improvement relative to the standalone INS is provided for each case. Similar to the mean position error, a 40% improvement rate is achieved, with a slight decrease as time progresses from ~43% at 30 sec. to 37%, or more, rate of improvement relative to the standalone INS.

Table 4: Mean velocity error obtained from five trajectories [m/s]

Time	INS	VC	VC+AVC	VC+ HC	VC+HC+AVC
30 [sec]	2.41	1.43 (42.2%)	1.43 (42.2%)	1.37 (43.5%)	1.35 (44.1%)
60 [sec]	6.73	4.30 (37.5%)	4.14 (39.1%)	4.31 (37.2%)	4.08 (39.3%)
90 [sec]	10.6	6.84 (37.0%)	6.81 (36.9%)	6.85 (37.1%)	6.16 (39.7%)

Considering the fact that both position and velocity errors were obtained from the five trajectories, it is clear that the EVC improves the standalone INS. The combined VC+AVC vehicle constraint obtained the overall best performance.

Next, we focus on trajectories 2, 3, and 4 which feature typical urban environment trajectories. We evaluate the level of improvement when incorporating the vehicle constraints in the EVC approach relative to the standalone INS and the regular integration approach. The results of the position and velocity errors and amount of improvements are summarized in Tables 5, and 6. In all the cases described in Table 5, the mean INS position error improved by more than 48% regardless of the duration. In addition, the newly proposed approach outperformed the classical one in all vehicle-constraint implementations and at all durations. This result reflects the improvement in modeling the dynamics in the EVC approach relative to the regular approach. As the underlying vehicle constraints assumptions in the examined trajectories were valid in most parts of the trajectories, the EVC implementation removed the corresponding IMU measurement noise form the system matrix, matching the filter to the actual physical behavior of the vehicle.

The greatest improvement relative to the standalone INS and the classical approach was at the 30 seconds period for all vehicle constraints, as expected. There the VC+AVC and VC+HC+AVC constraints obtained the best performance, improving the classical approach by 12.8%. For the velocity errors, the same behavior was observed where the EVC approach improved the standalone INS by more than 42% for all vehicle constraints at all time periods.

Table 5: Mean position error and improvements obtained from three trajectories

Time	Measure	VC	VC+AVC	HC+VC	HC+VC+AVC
30 [sec]	Error [m]	8.4	7.9	9.3	9.0
	Improvement relative to the standalone INS [%]	54.5	57.1	50.3	51.8
	Improvement relative to the classical impl. [%]	10	12.8	7.7	12.8

60 [sec]	Error	63.5	62.2	65.6	66.4
	Improvement relative to the standalone INS [%]	50.1	51.3	48	47.6
	Improvement relative to the classical impl. [%]	7.0	8.4	6.9	7.4
90 [sec]	Error	144.6	145.6	144.5	144.5
	Improvement relative to the standalone INS [%]	48.8	48.4	48.7	48.7
	Improvement relative to the classical impl. [%]	5.2	4.4	6.8	6.8

Table 6: Mean velocity error and improvements obtained from three trajectories

Time	Measure	VC	VC+AVC	HC+VC	HC+VC+AVC
30 [sec]	Error [m/s]	1.33	1.32	1.35	1.33
	Improvement relative to the standalone INS [%]	45.8	46.2	44.3	45.8
	Improvement relative to	6.1	4.0	9.3	15.1

	the classical impl. [%]				
60 [sec]	Error [m/s]	3.36	3.41	3.40	3.45
	Improvement relative to the standalone INS [%]	46.3	45.4	45.8	44.3
	Improvement relative to the classical impl. [%]	4.1	1.5	6.1	3.8
90 [sec]	Error [m/s]	5.40	5.56	5.53	5.34
	Improvement relative to the standalone INS [%]	43.9	42.1	43.1	44.4
	Improvement relative to the classical impl. [%]	4.0	0.0	6.51	7.5

EVC EVALUATION WITH AN ODOMETER

To further improve the INS aiding and utilize a fusion strategy which is common in many applications (e.g., [5]), an odometer was fused with the vehicle constraints. Trajectories 2, 3 and 4 are used for the comparison. Improvement rates relative to the standalone INS readings and the classical implementation are summarized in Tables 7 and 8.

The following observations regarding the fusion of the odometer with VC in the EVC approach can be drawn: 1) the EVC and odometer fusion greatly improved (more than 52%) the standalone INS performance for both position and velocity errors; 2) the odometer addition improved the performance of VC as a single aiding for both position and velocity errors; and 3) the proposed method obtained better performance compared to the classical approach, improving by more than 16% in position error and by more than 10% in the velocity error after 30 seconds. As can also be seen in both tables, for 90 seconds periods the contribution relative to the classical approach decreased, becoming almost negligible.

Generally, the fusion of odometer with the body vehicle constraint enables measurement of the whole velocity vector, thereby greatly improving the performance. In particular, in the EVC implementation, the corresponding IMU measurement noise was removed from the system matrix, matching the filter to the actual physical behavior of the vehicle and so improving performance of the regular implementation.

Table 7: Mean position error and improvements obtained from three trajectories

Time	Measure	VC	VC + Odometer
30 [sec]	INS Improvement [%]	54.5	81.1
	Classical Implementation Improvement [%]	10.0	16.1
60 [sec]	INS Improvement [%]	50.1	67.3
	Classical Implementation Improvement [%]	7.0	18.2

90 [sec]	INS Improvement [%]	48.8	59.7
	Classical Implementation Improvement [%]	5.2	1.1

Table 8: velocity error and improvements obtained from three trajectories

Time	Measure	VC	VC + Odometer
30 [sec]	INS Improvement [%]	45.8	64.4
	Classical Implementation Improvement [%]	6.1	10.6
60 [sec]	INS Improvement [%]	46.3	63.9
	Classical Implementation Improvement [%]	4.1	13.9
90 [sec]	INS Improvement [%]	43.9	52.1
	Classical Implementation Improvement [%]	4.0	0.3

CONCLUSIONS

This paper presented a methodology for enhancing vehicle constraints performance as aiding for MEMS INS over short periods of GPS outage (up to 90-seconds). The proposed approach was examined through five field experiments. Introduction of the vehicle constraints in the newly proposed approach outperformed the classical approach and reduced the navigation position and velocity errors.

The proposed implementation of the vehicle constraints requires no hardware change, yet only the addition of the proposed algorithm to the software is necessary.

REFERENCES

[1] Titterton D. H. and Weston J. L., Strapdown Inertial Navigation Technology – Second Edition, The American institute of aeronautics and astronautics and the institution of electrical engineers, 2004.

[2] Grewal M. S., Weill L. R. and Andrews A. P., Global positioning systems, inertial navigation and integration second edition, John Wiley & Sons, INC., Publications, 2007.

[3] Greenspan R. L., GPS and inertial navigation, in Global positioning system: theory and applications, Editores B. Parkinson, J. Spilker, P. Enge and P. Axelrad, AIAA, Washington, D. C., Vol. 2, 1996 , pp. 187-220.

[4] Kaplan E. D. and Hegarty C. J., Understanding GPS principles and applications second edition, Artech House, Boston, MA, 2006

[5] Stephen J. and Lachapelle G., Development and Testing of a GPS-Augmented Multi-Sensor Vehicle Navigation System, The journal of navigation, Vol. 54, No. 2, 2001, pp. 297-319.

[6] Godha, S., M. G. Petovello, and G. Lachapelle, Performance analysis of MEMS IMU/HSGPS/magnetic sensor integrated system in urban canyons, in Proceedings of ION GPS, Long Beach, CA, U. S. Institute of Navigation, Fairfax VA, September 2005, pp. 1977-1990.

[7] Tahk M. and Speyer J. L., Target Tracking Problems Subject to Kinematics Constraints, IEEE transactions on automatic control, Vol. 35, No. 3, 1990, pp. 324-326.

[8] Koifman M. and Bar-Itzhack Y., Inertial Navigation System Aided by Aircraft Dynamics, IEEE transactions on control systems technology, Vol. 7, No. 4, 1999, pp. 487-497.

[9] Brandit A. and Gardner J. F., Constrained navigation algorithm for strapdown inertial navigation systems with reduced set of sensors, Proceedings of the American control conference, Philadelphia PA., 1998, pp. 1848-1852.

[10] Dissanayake G., Sukkarieh S., Nebot E. and Durrant-Whyte H., The Aiding of a Low Cost Strapdown Inertial Measurement Unit Using Vehicle Model Constraints for Land Vehicle Applications, IEEE transactions on robotics and automation, Vol. 17, No. 5, 2001, pp. 731-747.

[11] Shin E.-H., Accuracy Improvement of Low Cost INS/GPS for Land Applications, UCGE reports number 20156, the University of Calgary, Calgary, Alberta, Canada, 2001

[12] Godha S. and M. E. Cannon, GPS/MEMS INS Integrated System for Navigation in Urban Areas, GPS Solutions, Vol. 11, 2007, pp. 193-203.

[13] Klein I., Filin S. and Toledo T., Pseudo-measurements as aiding to INS during GPS outages, NAVIGATION, Vol. 57, No. 1, 2010, pp. 25-34.

[14] Lachapelle, G., O. Mezentsev, J. Collin, and G. Macgougan, Pedestrian and vehicular navigation under signal masking using integrated HSGPS and self contained sensor technologies, 11th World Congress, International Association of Institutes of Navigation, 21-24 October, Berlin, 2003.

[15] Syad Z, Aggarwal P, Yang Y, and El-Sheimy N, Improved vehicle navigation using aiding with tightly coupled integration, IEEE Vehicular technology conference, 2008, pp. 2077-2081.

[16] Britting K. R., Inertial Navigation Systems Analysis, John Wily & Sons Inc., 1971.

[17] Microbotics website – www.microboticsinc.com

[18] Zarchan P. and Musoff H., Fundamentals of Kalman filtering: a practical approach second edition, The American Institute of Aeronautics and Astronautics, Inc. , Reston, Verginia, 2005.

[19] Maybeck P. S., Stochastic models, Estimation and Control, Volume 1, Navtech Book & Software store, 1994

APPENDIX - A

The following matrixes are associated with the INS state space error model Eq. (4)

$$F_{rr} = \begin{bmatrix} 0 & 0 & \dfrac{-V_N}{(M+h)^2} \\ \dfrac{V_E \sin(\phi)}{(N+h)\cos^2(\phi)} & 0 & \dfrac{-V_E \sin(\phi)}{(N+h)^2 \cos^2(\phi)} \\ 0 & 0 & 0 \end{bmatrix} \tag{A.1}$$

$$F_{rv} = \begin{bmatrix} \dfrac{1}{(M+h)} & 0 & 0 \\ 0 & \dfrac{1}{(N+h)\cos(\phi)} & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{A.2}$$

$$F_{vr} = \begin{bmatrix} -2V_E\omega_e\cos(\phi) - \frac{V_E^2}{(N+h)\cos^2(\phi)} & 0 & \frac{-V_N V_D}{(M+h)^2} + \frac{V_E^2\tan(\phi)}{(N+h)^2} \\ 2\omega_e\left(V_N\cos(\phi) - V_D\sin(\phi)\right) + \frac{V_E V_N}{(N+h)\cos^2(\phi)} & 0 & \frac{-V_N V_D}{(N+h)^2} - \frac{V_E V_N\tan(\phi)}{(N+h)^2} \\ 2V_E\omega_e\sin(\phi) & 0 & \frac{V_E^2}{(N+h)} + \frac{V_N^2}{(M+h)} - \frac{2\gamma}{(R+h)} \end{bmatrix} \quad \text{(A.3)}$$

$$F_{vv} = \begin{bmatrix} \frac{V_D}{(M+h)} & -2\omega_e\sin(\phi) - \frac{2V_E\tan(\phi)}{(N+h)} & \frac{V_N}{(M+h)} \\ 2\omega_e\sin(\phi) + \frac{V_E\tan(\phi)}{(N+h)} & \frac{V_D + V_N\tan(\phi)}{(N+h)} & 2\omega_e\cos(\phi) + \frac{2V_E}{(N+h)} \\ -\frac{2V_N}{(M+h)} & -2\omega_e\cos(\phi) - \frac{2V_E}{(N+h)} & 0 \end{bmatrix} \quad \text{(A.4)}$$

$$F_{\varepsilon r} = \begin{bmatrix} -\omega_e\sin(\phi) & 0 & \frac{-V_E}{(N+h)^2} \\ 0 & 0 & \frac{V_N}{(M+h)^2} \\ -\omega_e\cos(\phi) - \frac{V_E}{(N+h)\cos^2(\phi)} & 0 & \frac{V_E\tan(\phi)}{(N+h)^2} \end{bmatrix} \quad \text{(A.5)}$$

$$F_{\varepsilon v} = \begin{bmatrix} 0 & \frac{1}{(N+h)} & 0 \\ -\frac{1}{(M+h)} & 0 & 0 \\ 0 & \frac{-\tan(\phi)}{(N+h)} & 0 \end{bmatrix} \quad \text{(A.6)}$$

$$F_{\varepsilon\varepsilon} = \begin{bmatrix} 0 & \omega_e\sin(\phi) + \frac{V_E\tan(\phi)}{(N+h)} & \frac{V_N}{(M+h)} \\ -\omega_e\sin(\phi) - \frac{V_E\tan(\phi)}{(N+h)} & 0 & -\omega_e\cos(\phi) - \frac{V_E}{(N+h)} \\ -\omega_e\cos(\phi) - \frac{V_E}{(N+h)\cos^2(\phi)} & \omega_e\cos(\phi) + \frac{V_E}{(N+h)} & 0 \end{bmatrix} \quad \text{(A.7)}$$

where $v^n \triangleq \begin{bmatrix} v_N & v_E & v_D \end{bmatrix}^T$ is the velocity vector in the n-frame and the rest of the parameters were defined in the text.

APPENDIX – B

In general, a Kalman filter algorithm involves two steps: i) prediction of the state based on the system model, and ii) update of the state based on the measurements. The covariance associated with the prediction step is given by [18]:

$$\hat{x}_{k+1}^{-} = \Phi \hat{x}_k^{+}, \Phi = e^{F(t)\Delta t} \tag{B.1}$$

$$P_{k+1}^{-} = \Phi P_k^{+} \Phi^T + Q_k \tag{B.2}$$

where the superscripts – and + represent the predicted and updated quantities (before and after the measurement update, respectively); x and P are the system state and the associated error covariance matrices respectively; Φ is the state transition matrix from time k to time k+1; $F(t)$ is the system dynamics matrix; and Q_k is the process-noise covariance-matrix [19] given by:

$$Q_k \approx \frac{1}{2}\left[\Phi_k G(t_k) Q(t_k) G^T(t_k) + G(t_k) Q(t_k) G^T(t_k) \Phi_k^T\right]\Delta t \tag{B.3}$$

where, $G(t)$ is the shaping matrix. Δt is the time step. The second step is the measurement update:

$$K_{k+1} = P_{k+1}^{-} H_{k+1}^T \left(H_{k+1} P_{k+1}^{-} H_{k+1}^T + R_{k+1}\right)^{-1} \tag{B.4}$$

$$\hat{x}_{k+1}^{+} = \hat{x}_{k+1}^{-} + K_{k+1}\left(z_{k+1} - H_{k+1}\hat{x}_{k+1}^{-}\right) \tag{B.5}$$

$$P_{k+1}^{+} = \left(I - K_{k+1}H_{k+1}\right)P_{k+1}^{-} \tag{B.6}$$

where K_k is the Kalman gain, H_k is the measurement matrix, R_k is the measurement noise covariance matrix, and z_k is the measurement.