State Dependence in Lane-Changing Models

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Lane changes have a substantial impact on traffic flow characteristics. The lane-changing model is therefore an important element in microscopic traffic simulations. Lane changing is commonly modeled in two steps: lane choice, which captures the desire to change lanes, and the decision about whether a desired lane change can be completed, which is captured by gap acceptance models. Most current models assume that these decisions are repeated at every time step of the simulation independently of previous decisions. However, it may be more realistic to assume that drivers persist in their lane choices, and so their desired lane at any time point depends on earlier choices. To capture persistency in lane-changing behavior, a model that integrates a hidden Markov model (HMM) structure is presented. The evolution of lane choices, which are the underlying hidden states, is modeled using a Markovian process. The observed lane-changing actions depend on these hidden lane choices. An important difficulty that arises with this model structure is the problem of unobserved initial conditions on the hidden states. A method to address this problem is proposed. Estimation results of the resulting model are presented and compared with a model that does not incorporate state dependence.

Lane changes have a substantial impact on traffic flow characteristics. The lane-changing model is therefore an important element in microscopic traffic simulations. Lane changing is commonly modeled in two steps: lane choice, which captures the desire to change lanes, and the decision about whether a desired lane change can be completed, which is captured by gap acceptance models. Modeling the lane-changing decision process is complex because of the many factors a driver considers before making a decision. Furthermore, the decision process is latent in nature, with the only observable part being the lane change action, if any, that the driver executes. The desire to change lanes that underlies these actions cannot be observed.

Most current lane-changing models assume that lane choices are affected by two basic considerations: gaining speed or queue advantage and being in the correct lanes to follow the vehicle's path. Thus, lane changes are often broadly classified as either mandatory or discretionary [e.g., Gipps (*1*), Halati et al. (*2*), Zhang et al. (*3*), Hidas and Behbahanizadeh (*4*), Ahmed (*5*), Hidas (*6*), and Barcelo and Casas (*7*)]. Drivers undertake mandatory lane changes when they must leave their current lane to follow their travel path, to bypass a lane blockage, or to comply with traffic signs. They perform discretionary lane changes when they perceive that they can improve their driving conditions by moving to another lane although it is not necessary to do so. Toledo et al. (*8, 9*) proposed an integrated lane choice model that allows joint evaluation of mandatory and discretionary considerations and captures trade-offs between these considerations. The model in Toledo et al. consists of two levels: an explicit choice of a target lane and gap acceptance decisions (*9*). The direction for an immediate lane change is dictated by the target lane choice. The model proposed in this paper adopts and extends this model structure. Other directions of improvement to lane-changing models have been proposed, including introduction of models that capture additional lane-changing mechanisms, such as cooperation and forcing [e.g., Hidas (*10*) and Toledo et al. (*11*)], to better represent behavior in heavy congestion or macroscopic lane changing rate models [e.g., Laval and Daganzo (*12*)].

Although the models listed above differ in their specification details, they share an important limitation in that they assume that drivers' choices are instantaneous and independent of those they made earlier. Lane changes are modeled as discrete events occurring at specific points in time, with the decision process being repeated at every time step of the simulation. However, it may be more realistic to assume that drivers persist in their lane choices, and so their target lane choice at any time point may depend on earlier ones.

This paper develops a framework and presents estimation results for a lane-changing model that explicitly takes into account correlations and dependencies in the lane-changing decisions drivers make over time. To that end, the model incorporates two mechanisms: (*a*) an individual-specific error term in all components of the model captures correlations among the decisions made by the same driver across choices and over time and (*b*) a state-dependence formulation captures persistence and inertia in driving choices through the impact of earlier choices on later ones. The state dependence is assumed in the latent desired lane choices. A hidden Markov model (HMM) is used to model this process.

The rest of this paper is organized as follows: the next section briefly introduces the theory of HMMs and their application in the field of driving behavior. The following section presents the integration of the HMM structure within lane-changing models and the detailed formulation of the resulting model. Next, the data used for model estimation and the likelihood function derived for these data are presented. That is followed by the estimation results and comparison with similar models that do not incorporate state dependency. Finally a summary and discussion are presented.

HIDDEN MARKOV MODELS

Markov models are well accepted to model the dynamics of systems that can transition between finite numbers of states. The assumption of these models is that at each time step the system may remain in its current state or change to another. The transition from state *i* to

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state j occurs with a known transition probability p_{ij}. These probabilities do not depend on any previous states the system was in before the current one. Thus, every future state is conditionally independent of every previous state.

However, there may be cases in which the system state is not directly observable and only the outcome of another process that probabilistically depends on the system state is observed. HMMs are used to model these cases (*13*). HMMs assume that there exists a latent process that transitions the system from state to state and that this process could be studied using the observable outputs of another process that is affected by the underlying system state. The observable output is a realization of random variables with a density function that depends on the hidden state. Mathematically, the HMM is defined by (Π, A, B). Π is a vector of the probabilities π_i that initially the system is in state i. A is the state transition matrix with entries $a_{ji} = p(x_i|x_j)$ that represent the probability of moving to hidden state i from hidden state j. B is the confusion matrix that contains the probabilities $b_{ik} = p(y_k|x_i)$ of observing outcome k when the hidden system state is i. Note, that in the classic model, the matrices A and B are time independent. That is, the probabilities do not change in time as the system evolves.

To illustrate the HMM structure, Figure 1 shows the state transition diagram of an HMM. Each of the two hidden states (x_i) maps to one of the three observable outcomes (y_k) with some probability (b_{ik}). The state transition probabilities (a_{ji}) are the probabilities of moving from one hidden state to another.

A few applications of HMMs have been reported in the field of driving behavior, mainly in the context of behavior recognition in advanced driving assistance systems. In these applications HMMs are used to identify actions and maneuvers that drivers intend to undertake using observations from various sensors in the vehicle. For example, Pentland and Liu used information on changes in the vehicle heading and acceleration as the observable outcomes of a model that aimed to identify the hidden intended actions of the driver (e.g., stop, turn, change lanes, overtake) during the first few preparatory steps of the maneuver (*14*). Kuge et al. used a similar framework to predict the hidden lane change intentions of drivers (*15*). Data on the steering angle, steering angle velocity, and steering force were the observable outcomes. Dapzol proposed a model that identifies 40 different driving situations as hidden states using outcomes measured in the steering wheel, clutch, brake, and accelerator pedals (*16*).

An application closer to the present work was presented by Zou and Levinson (*17*). The observable outcomes are defined as combinations of data on changes in speed (acceleration, deceleration, or cruising) and on whether the vehicle is in conflict with other vehicles to study the hidden attitudes toward traffic conditions of drivers that approach an intersection. The hidden states were defined using clustering analysis and meant to capture the heterogeneity in the driver population. In the case study presented, the HMM fitted the data better than simpler models in estimation and in prediction of drivers' behavior. But the authors do not provide any interpretation of the resulting hidden clusters.

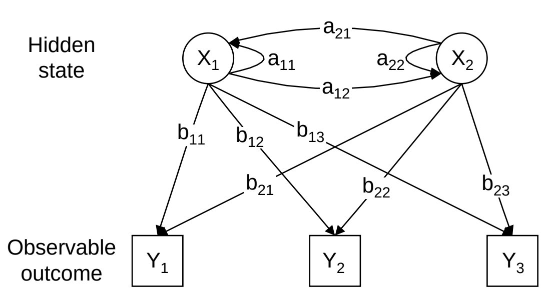

FIGURE 1 State transition diagram of a hidden Markov model.

In summary, most applications of HMM in driving modeling focused on identifying the intention of the driver using observed behavior (i.e., acceleration, steering angle). These models can, for example, detect whether a driver is changing lanes at the time of the observation but do not attempt to explain why the lane change is undertaken and so cannot predict lane changes ahead of time. They therefore cannot be used for traffic simulations. In this paper, an HMM formulation is incorporated within a lane-changing model to capture persistence in lane-changing decisions drivers make and through that improve the ability to predict the occurrence of lane changes.

INTEGRATION OF HMM IN A LANE-CHANGING MODEL

Overall Structure

As noted above, the lane change decision process is assumed to have two steps: the target lane choice and acceptance of a gap in the direction of the target lane. The target lane is the lane the driver perceives as the best choice. This decision process is latent because the target lane choice is unobservable. For example, it may be observed that a driver stays in the current lane, but the reason that caused the driver to stay there cannot be observed. The driver may have chosen not to pursue a lane change at all or may have chosen to move to another lane but could not complete the lane change. In the terminology of HMM the target lane selection can be viewed as the dynamic latent process that evolves from state to state. The observable outcome is the lane the vehicle is in at any time step. The process that relates the underlying hidden states (target lane choices) to the lane observations is the gap acceptance function.

Figure 2 demonstrates the structure of the resulting model. In the figure ovals represent latent states and rectangles represent observed outcomes. The decision process is shown for a vehicle that is currently in Lane 2 (lane numbers are ordered from right to left) and that previously chose Lane 3 as the target lane. The arrows from Lane 3 to all lanes indicate the dependence of the target lane choice probabilities on the previous target lane choice. The target lane choice determines the choice of the immediate change direction the driver will consider. The driver then evaluates the available gap in the adjacent lane in this direction and either accepts it and changes lanes or rejects it and does not change lanes. Thus, the observable lane outcome depends on the hidden target lane the driver chose.

The target lane choice and gap acceptance decisions are affected not only by the hidden states but also by variables that capture the driver's tactical driving goals and personal characteristics and the conditions in the neighborhood of the vehicle. In the next sections the specification of the two submodels to capture these effects are presented.

Target Lane Model

The target lane model assumes that the driver chooses a target lane among all the available lanes in the roadway. Each lane has an associated utility to the driver, and the lane with highest utility is chosen.

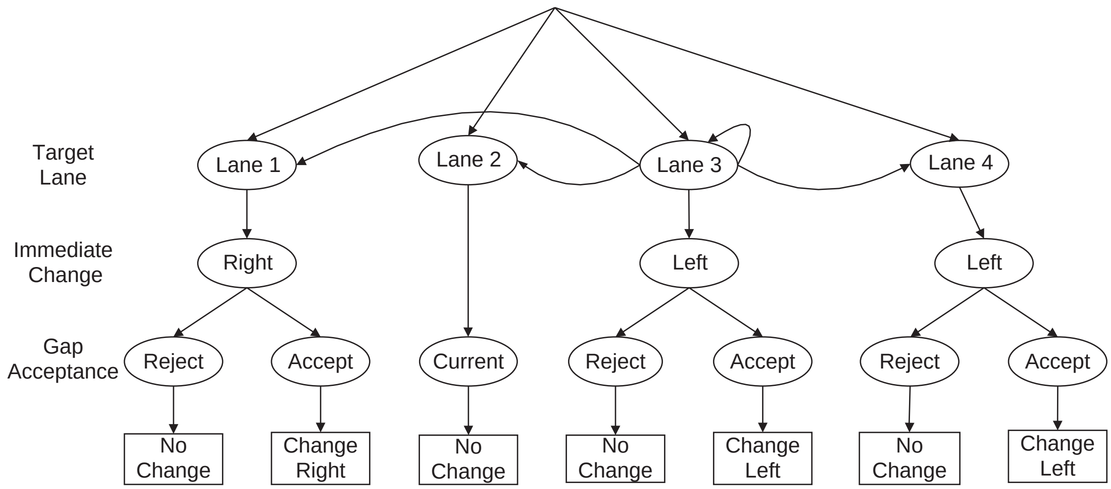

FIGURE 2 Structure of proposed lane-changing model.

Lane utilities may depend on explanatory variables, such as those that capture the conditions in the immediate neighborhood in each lane (e.g., leader speeds in each lane and presence of heavy vehicles) and path plan considerations (e.g., the distance and number of required lane changes to a point where the driver must be in specific lanes to follow the path). In most cases information about the characteristics of drivers and their vehicles (e.g., aggressiveness, driving skill, vehicle's speed and acceleration capabilities) is not available. Therefore, an individual-specific error term is introduced in the utility model to capture the unobserved characteristics of the driver. The resulting utility of being in lane i to driver n at time t is given by

$$U_{nt}^{i} = \beta^{i} X_{nt}^{i} + \rho\delta_{n,t-1}^{i} + \alpha^{i}\upsilon_{n} + \epsilon_{nt}^{i} \tag{1}$$

where

U_{nt}^{i} = utility of lane i to individual n at time t;
X_{nt}^{i} and β^{i} = vector of explanatory variables and the corresponding parameters, respectively;
$\delta_{n,t-1}^{i}$ = dummy variable = 1 if lane i is the target lane at time t–1 and 0 otherwise (this variable captures the dependence of the current target lane choice on the one in the previous time step);
ρ = parameter of this variable, which captures the strength of the state dependence in target lane choices (it can be interpreted as a measure of drivers' persistence in their target lane choices);
υ_{n} and α^{i} = individual-specific error term and the associated parameter, respectively; and
ϵ_{nt}^{i} = generic random term.

Assuming that ϵ_{nt}^{i} are independently and identically Gumbel distributed and that ϵ_{nt}^{i} and υ_{n} are independent of each other, the target lane choice probabilities for the various lanes, conditional on υ_{n} and the previous target lane choice, are given by

$$P\left(\mathrm{TL}_{nt} = i \middle| \mathrm{TL}_{n,t-1}, \upsilon_{n}\right) = \frac{\exp\left(\beta^{i} X_{nt}^{i} + \rho\delta_{n,t-1}^{i} \middle| \upsilon_{n}\right)}{\sum_{j} \exp\left(\beta^{j} X_{nt}^{j} + \rho\delta_{n,t-1}^{j} \middle| \upsilon_{n}\right)} \tag{2}$$

where TL_{nt} is the target lane for driver n at time t.

Gap Acceptance Model

The choice of the target lane dictates the direction for lane change (right, left, or none). If a lane change is required, the driver evaluates the available gap in the adjacent lane in this direction. The available gap is defined by the lead and lag vehicles in the adjacent lane. The lead gap is the clear spacing between the rear of the lead vehicle and the front of the subject vehicle. Similarly, the lag gap is the clear spacing between the rear of the subject vehicle and the front of the lag vehicle. One or both of these gaps may be negative if the vehicles overlap.

The model assumes that the available lead and lag gaps are compared with the corresponding critical gaps, which are the minimum acceptable gaps. Therefore, an available gap is accepted only if it is greater than the critical gap. Critical gaps vary for different individuals and with the situation. The critical gaps are therefore modeled as random variables whose means are functions of explanatory variables and incorporate the individual-specific error term to capture unobserved driver characteristics. An exponential functional form is used to ensure that critical gaps are always positive:

$$\ln\left(\mathrm{CG}_{nt}^{g,\mathrm{TL}}\right) = \beta^{g} X_{nt}^{g,\mathrm{TL}} + \alpha^{g}\upsilon_{n} + \epsilon_{nt}^{g} \qquad g \in \{\text{lead, lag}\} \tag{3}$$

where

$\mathrm{CG}_{nt}^{g,\mathrm{TL}}$ = critical gap g in adjacent lane in direction of target lane;
$X_{nt}^{g,\mathrm{TL}}$ and β^{g} = vector of explanatory variables and the corresponding parameters, respectively;
α^{g} = parameter of the individual-specific random term; and
ϵ_{nt}^{g} = generic error term.

The gap acceptance model assumes that the lead gap and the lag gap must be acceptable for the vehicle to change lanes. The probability of changing lanes, conditional on the individual-specific term and the target lane, is therefore given by

$$\begin{aligned} &P\left(\text{change to target lane} \middle| \mathrm{TL}_{nt}, \upsilon_{n}\right) = P\left(l_{nt}^{\mathrm{TL}} = 1 \middle| \mathrm{TL}_{nt}, \upsilon_{n}\right) = \\ &P\left(\text{accept lead gap} \middle| \mathrm{TL}_{nt}, \upsilon_{n}\right) P\left(\text{accept lag gap} \middle| \mathrm{TL}_{nt}, \upsilon_{n}\right) = \\ &P\left(G_{nt}^{\text{lead, TL}} > \mathrm{CG}_{nt}^{\text{lead, TL}} \middle| \mathrm{TL}_{nt}, \upsilon_{n}\right) \cdot P\left(G_{nt}^{\text{lag,TL}} > \mathrm{CG}_{nt}^{\text{lag,TL}} \middle| \mathrm{TL}_{nt}, \upsilon_{n}\right) \end{aligned} \tag{4}$$

where $G_{nt}^{\text{lead,TL}}$ and $G_{nt}^{\text{lag,TL}}$ are the available lead and lag gaps, respectively. l_{nt}^{TL} is an indicator to the lane changing outcome, which takes the value 1 if the vehicle changes lanes in the target lane direction and 0 otherwise.

Assuming that $\epsilon_{nt}^{g} \sim N(0, \sigma_g^2)$, the conditional probability that a gap is acceptable is given by

$$P\left(G_{nt}^{g,\text{TL}} > \text{CG}_{nt}^{g,\text{TL}} \middle| \text{TL}_{nt}, \upsilon_n\right) = P\left(\ln\left(G_{nt}^{g,\text{TL}}\right) > \ln\left(\text{CG}_{nt}^{g,\text{TL}}\right) \middle| \text{TL}_{nt}, \upsilon_n\right) = \Phi\left[\frac{\ln\left(G_{nt}^{g,\text{TL}}\right) - \left(X_{nt}^{g,\text{TL}}\beta^g + \alpha^g \upsilon_n\right)}{\sigma_g}\right] \quad (5)$$

where $\Phi[\bullet]$ denotes the cumulative standard normal distribution.

MODEL ESTIMATION

Data

The data set of vehicle trajectories used to estimate the lane changing model was collected in a four-lane section of I-395 southbound in Arlington, Virginia, using video cameras (*18*). The section is shown schematically in Figure 3. This data set is particularly useful for estimation of the model because of the geometric characteristics of the site: the site is 997 m long with two off-ramps and an on-ramp and therefore includes weaving sections that allow capturing the effect of the path plan on driving behavior.

The data set contains observations of the position, lane, and dimensions of every vehicle in the section every 1 s. Explanatory variables required by the model, such as relations between the subject and other vehicles (e.g., relative speeds, time and space headways), were inferred from the raw data set. The data used for model estimation include 442 vehicles for a total of 15,632 observations. On average a vehicle was observed for 35.4 s (observations). All vehicles are first observed at the upstream end of the freeway section. At the downstream end 76% of the vehicles stayed on the freeway and 8% and 16% used the first and second off-ramps, respectively. Observed speeds ranged from 0.4 to 25.0 m/s., with a mean of 15.6 m/s. Densities ranged from 14.2 to 55.0 veh/km/lane, with a mean of 31.4 veh/km/lane. The level of service in the section ranged from D to E.

Likelihood Function

The path plan has an important impact on drivers' lane choices. It is captured by variables such as the distance to the point where the driver needs to be in certain lanes to follow the path. However, this information is not known for vehicles that exit the freeway downstream of the observed section. These distances are therefore modeled as latent variables. A discrete distribution of the distances from the downstream end of the observed section to the exit points, based on the locations of downstream off-ramps, is used. The probability mass function of distances beyond the downstream end of the section to the off-ramps used by drivers is given by

$$p(d_n) = \begin{cases} \pi_1 & \text{first downstream exit } \left(d^1\right) \\ \pi_2 & \text{second downstream exit } \left(d^2\right) \\ 1 - \pi_1 - \pi_2 & \text{otherwise } \left(d^3\right) \end{cases} \quad (6)$$

where, π_1 and π_2 are parameters that represent the proportions of drivers using the first and second downstream off-ramp, respectively. d^1, d^2, and d^3 are the distances beyond the downstream end of the section to the first, second, and subsequent exits, respectively. For the subsequent exits an infinite distance is assumed ($d^3 = \infty$), which implies that drivers that use these exits ignore path plan considerations in their lane choice.

The joint probability density of a combination of the lane change outcome observed for driver n at time t and the target lanes at time t and t–1, conditional on the individual specific variables, υ_n, and the distance to the exit point, d, is given by

$$P\left(\text{TL}_{nt}, \text{TL}_{n,t-1}, l_{nt}^{\text{TL}} \middle| d_n, \upsilon_n\right) = P\left(\text{TL}_{nt} \middle| \text{TL}_{n,t-1}, d_n, \upsilon_n\right) P\left(l_{nt} \middle| \text{TL}_{nt}, \upsilon_n\right) P\left(\text{TL}_{n,t-1} \middle| d_n, \upsilon_n\right) \quad (7)$$

where $P(\text{TL}_{nt} | \text{TL}_{n,t-1}, d_n, \upsilon_n)$ and $P(l_{nt}^{\text{TL}} | \text{TL}_{nt}, \upsilon_n)$ are given by Equations 2 and 4, respectively. $P(\text{TL}_{n,t-1} | d_n, \upsilon_n)$ is calculated recursively as follows:

$$P\left(\text{TL}_{n,t-1} = i \middle| d_n, \upsilon_n\right) = \sum_{j \in J} P\left(\text{TL}_{n,t-1} = i \middle| \text{TL}_{n,t-2} = j, d_n, \upsilon_n\right) P\left(\text{TL}_{n,t-2} = j, d_n, \upsilon_n\right) \quad (8)$$

where J is the set of alternative lanes in the section.

Given the initial target lane probabilities, $P(\text{TL}_{n,0} | d_n, \upsilon_n)$, these values can be calculated for any t. The lane-changing outcomes of driver n are observed over a sequence of T consecutive time intervals. With the assumption that conditional on d_n and υ_n these observations

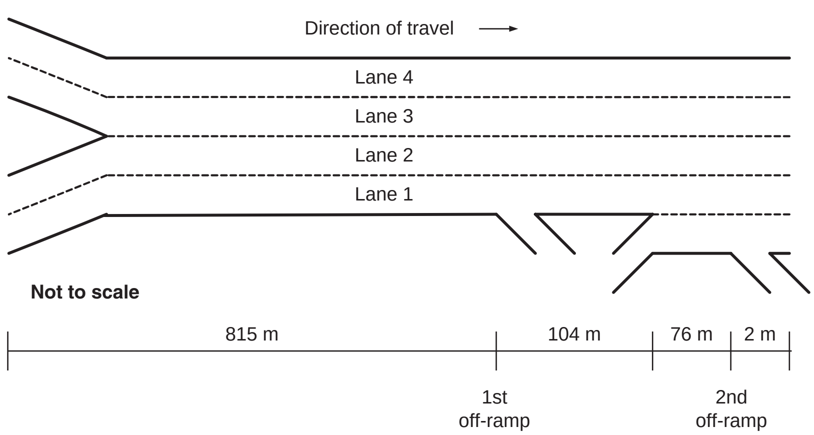

FIGURE 3 I-395 data collection site.

are independent, the joint probability of the sequence of observations is given by

$$P\left(\mathbf{l}_n \middle| d_n, \upsilon_n\right) = \prod_{t=1}^{T} \sum_i \sum_j P\left(\mathrm{TL}_{nt} = i, \mathrm{TL}_{n,t-1} = j, l_{nt}^i \middle| d_n, \upsilon_n\right) \tag{9}$$

where, $\mathbf{l}_n$ is the sequence of lane changing outcomes. The dependence on the initial conditions stems from the recursive calculation of $P_n(\mathrm{TL}_{t-1}|d_n, \upsilon_n)$.

The unconditional individual likelihood function is obtained by integrating (or summing, for the discrete variable d_n) the conditional probability over the distributions of the latent variables:

$$L_n = \int_{\upsilon} \sum_d P\left(\mathbf{l}_n \middle| d_n, \upsilon_n\right) p(d)\, f(\upsilon) d\upsilon \tag{10}$$

where $p(d)$ is given by Equation 6 and $f(\upsilon)$ is the standard normal probability density function.

Assuming that observations of different drivers are independent, the log likelihood function for all N individuals in the sample is given by

$$L = \sum_{n-1}^{N} \ln\left(L_n\right) \tag{11}$$

The maximum likelihood estimates of the model parameters are found by maximizing this function.

INITIAL CONDITIONS

The state dependence creates dependence on the initial hidden states. However, these initial conditions are not observable. Furthermore, they do not represent the true starting point of the process (the initial observation point is dictated by the data collection system) and so are not fixed, but generated by the dynamic process, which depends on the individual-specific terms. Therefore, the initial conditions are not exogenous to the model. Treating them as exogenous generally results in inconsistent parameter estimates.

To overcome this difficulty an approach proposed by Heckman for dynamic discrete choice models with state dependence was adapted to the present problem with the hidden Markov assumption (*19*). In this approach the distribution of the initial conditions, conditional on the individual-specific error term, is approximated. Specifically, for the initial observation for each driver in the sample the utility functions are approximated by a reduced form:

$$U_{n,0}^i = \beta^{0,i} X_{n,0}^i + \alpha^{0,i} \upsilon_n + \epsilon_{n,0}^i \tag{12}$$

where $U_{n,0}^i$ is the utility of target lane i to driver n at time $t = 0$. $\beta^{0,i}$ are the reduced form model parameters, which are not restricted to equal those used in the model for the other time steps (Equation 1). $\alpha^{0,i}$ is the parameter of the individual-specific error term. It is also allowed to differ from the one used in other time steps, and so allows correlation between the utilities in the initial observation and subsequent ones to assume any value. $\epsilon_{n,0}^i$ is a generic error term.

The resulting model is estimated with the additional parameters for the utilities of the target lanes in the initial observations. This substantially increases the number of parameters to be estimated. But these additional parameters are introduced only for consistent model estimation. They are not used when the model is applied to predict driving behavior, for example in microscopic traffic simulation.

ESTIMATION RESULTS

All parameters of the state dependency lane changing model were estimated jointly using the likelihood function presented above. First, all parameters in the initial conditions model were allowed to differ from those used for the other observations. However, many of these parameters did not differ significantly, and so were restricted to have the same values. The results of this more parsimonious model are presented in Table 1.

Target Lane Model

Estimation results for the target lane model show that different types of variables affect target lane choices. These include lane-specific attributes, variables that capture driving conditions in the

TABLE 1 Estimation Results of Lane-Changing Model

Variable	Parameter	t-Statistic
Target Lane Model		
Lane attributes		
Lane 1 constant	–1.859	–3.43
Lane 2 constant	–0.649	–2.034
Lane 3 constant	–0.034	–0.18
Current lane dummy	3.264	14.10
Two or more lane changes from current lane	–4.132	–1.97
Driving neighborhood		
Front vehicle spacing (m)	0.026	4.09
Relative front vehicle speed (m/s)	0.134	2.67
Path plan		
Distance to exit and number of lane changes required	–2.604	–5.98
Next exit dummy, lane change(s) required	–1.624	–3.04
Remaining distance power, θ_{MLC}	–1.283	–2.67
Probability of taking 1st exit, π_1	0.0002	0.01
Probability of taking 2nd exit, π_2	0.047	2.044
Heterogeneity (individual-specific error coefficients)		
Lane 1, α^1	1.143	3.02
Lane 2, α^2	0.270	0.96
Lane 3, α^3	1.803	6.46
Lane 4, α^4	0.453	1.84
State dependency		
Persistence dummy, ρ	0.131	4.53
Initial conditions		
Initial current lane dummy	4.804	1.84
Initial path plan impact, two or more lane changes required to the exit	–1.309	–1.99
Initial front vehicle spacing (m)	–0.017	–1.92
Lead Critical Gap		
Constant	1.706	6.03
Relative lead speed positive, $\max(\Delta V_{\mathrm{nt}}^{\mathrm{lead}}, 0)$ (m/s)	–6.323	–3.31
Relative lead speed negative, $\min(\Delta V_{\mathrm{nt}}^{\mathrm{lead}}, 0)$ (m/s)	–0.155	–2.51
Heterogeneity coefficient of lead gap, α^{lead}	0.099	0.35
Standard deviation of lead gap, σ^{lead}	0.939	4.18
Lag Critical Gap		
Constant	1.429	5.63
Relative lag speed positive, $\max(\Delta V_{\mathrm{nt}}^{\mathrm{lag}}, 0)$ (m/s)	0.512	5.84
Heterogeneity coefficient of lag gap, α^{lag}	0.211	1.27
Standard deviation of lag gap, σ^{lag}	0.775	5.87

neighborhood of the vehicle, and variables that relate to the path plan. With respect to the model structure, the use of the individual-specific error and the state dependency formulation is justified by the results. A detailed discussion of the variables used and the specification of the systematic part of the target lane model was presented in previous works that used similar models (*9*). The focus of this paper is on the inclusion of state dependence in these models. Nevertheless, a brief discussion of the important estimation results is presented.

The estimated values of the lane-specific constants imply that, everything else being equal, the rightmost lane, Lane 1, is the most undesirable and that lanes to the left are increasingly more attractive. The estimated parameter values for the current lane dummy and the dummy variable for two or more lane changes that are required from the current lane indicate that drivers strongly prefer to stay in their current lane and to avoid making lane changes.

Driving conditions in the immediate neighborhood of the vehicle are reflected by two variables that capture the interactions of the vehicle with vehicles around it: the relative speed and the spacing with respect to the vehicles in front in the current or adjacent lanes. The signs of these parameters are positive, thus the utility of the current lane increases with the speed of the front vehicle and with the spacing between the two vehicles. The utilities of adjacent lanes also increase with the speed of the lead vehicles in these lanes.

The path plan impact variables indicate that the utility of a lane decreases with the number of lane changes that the driver needs to undertake to follow the intended path. This effect is magnified as the distance to the off-ramp decreases ($\theta^{MLC} = -1.283$, see Equation 13).

The parameters of the individual-specific error term (α^1, α^2, α^3, and α^4) capture the effects of unobserved driver characteristics on the target lane choice, thus accounting for correlations between observations of the same individual. α^1 and α^3 are more positive compared with α^2 and α^4. These are the rightmost lanes of the two freeways that merge at the upstream end of the section. Therefore, the individual-specific error term υ can be interpreted as positively correlated to the driver's timidity. A timid driver (i.e., $\upsilon_n > 0$) is more likely to choose the right lanes over the left compared with a more aggressive driver.

The state-dependency coefficient, ρ, is positive and significant. The utility of a lane increases if it is chosen as the target lane in the previous period. This implies that drivers are persistent in selecting their short-term lane-changing plan. To illustrate the impact of this variable, consider a case of a four-lane road, in which the vehicle is currently in Lane 2. Further assume that the lane attributes of all lanes are equal. Figure 4 shows the probabilities of choosing Lane 2 as the target lane depending on the target lane in the previous time step. The probability of choosing the lane is highest if it was also chosen in the previous time step and lower if another lane was chosen. Furthermore, the probability of choosing a lane is higher if the previously chosen lane was generally less attractive (i.e., had a lower probability of being chosen). The probability of choosing Lane 2 is the highest if the driver also decided to stay in this lane in the previous time period. However, if the previously chosen lane is not Lane 2, the probability of choosing Lane 2 in the current time period is highest if Lane 4 was previously chosen and lowest if Lane 3 was previously chosen. This is also the reverse order of the probabilities of choice of these lanes. Thus, the model predicts that the probability that the driver will persist with a previously chosen target lane increases with its quality. If the driver chose a weak plan (one that has a low choice probability), the probability of aborting it and choosing another one is higher compared with when the previously selected plan is strong.

To demonstrate further the effect of the state dependency on the behavior dynamics, consider a situation in which a driver in Lane 2 needs to change to Lane 1 to use an off-ramp and follow the path. Figure 5 shows the predicted probabilities of choosing Lane 1 for the target lane as a function of the distance to the off-ramp, for drivers that chose Lane 1 in the previous time step and for those that did not. In both cases, the probability of choosing Lane 1 is very low when the vehicle is far from the off-ramp, gradually increases as it

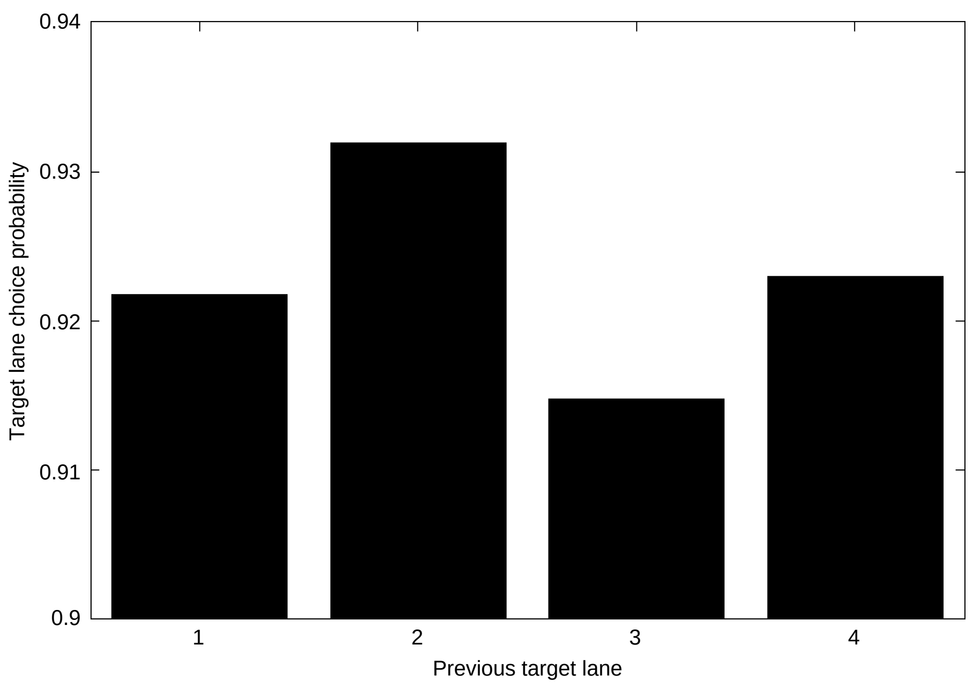

FIGURE 4 Target lane choice probabilities depending on previously chosen lane.

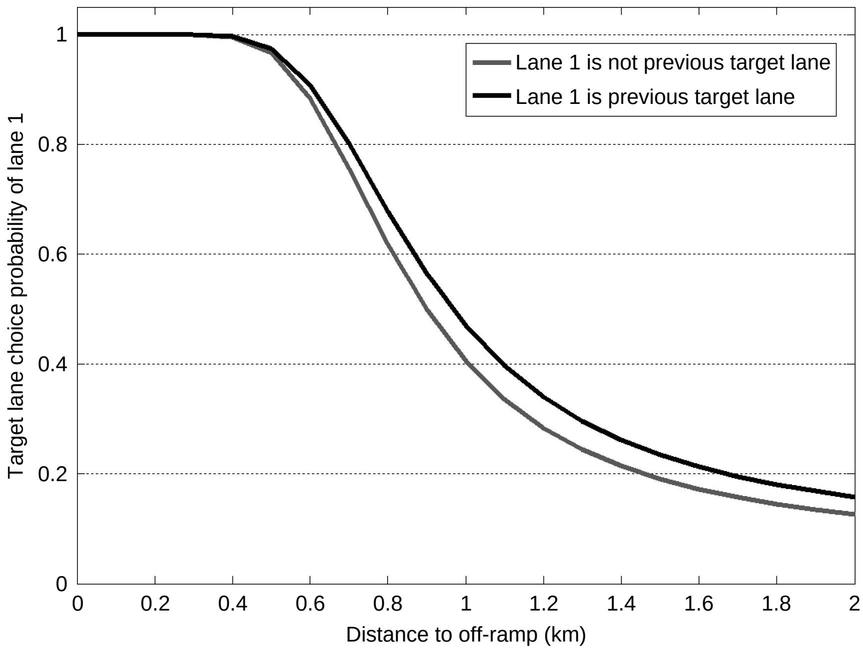

FIGURE 5 Predicted probabilities of choosing target lane closest to off-ramp used in path.

nears the exit, and asymptotically approaches a unit when the vehicle is very close to the exit point. However, the predicted probability of choosing Lane 1 is higher if the driver has already chosen this lane in the previous time period at any distance from the off-ramp. The figure also demonstrates that accounting for the state dependence in the behavior generates variability in the dynamics of the lane selection process among drivers.

In summary, the target lane utilities are given by

$$\begin{aligned} U^{i}_{nt} &= \beta^{i} + 0.026\Delta X^{i,\text{front}}_{nt}\delta^{i,\text{CL}}_{nt} + 0.134\Delta S^{i,\text{front}}_{nt}\delta^{i,\text{adj/CL}}_{nt} \\ &\quad + 3.264\delta^{i,\text{CL}}_{nt} - 4.132\delta^{i,\Delta\text{CL}\geq 2}_{nt} - 2.604\left[d^{\text{exit}}_{nt}\right]^{-1.283}\Delta\text{exit}^{i} \\ &\quad - 1.624\delta^{\text{next exit}}_{nt}\delta^{i,\Delta\text{exit}\geq 1}_{nt} + 0.131\delta^{i}_{n,t-1} + \alpha^{i}v_{n} + \epsilon^{i}_{nt} \end{aligned} \tag{13}$$

where

- β^i = constant of lane *I;*
- $\Delta X^{i,\text{front}}_{nt}$ and $\Delta S^{i,\text{front}}_{nt}$ = spacing and relative speed of front vehicle in lane *i,* respectively;
- $\delta^{i,\text{adj/CL}}_{nt}$ = indicator with value 1 if *i* is the current or an adjacent lane, and 0 otherwise;
- $\delta^{i,\text{CL}}_{nt}$ = 1 if *i* is the current lane, and 0 otherwise;
- $\delta^{i,\Delta\text{CL}\geq 2}_{nt}$ = indicator that takes value 1 if two or more lane changes are required from current lane to lane *i,* and 0 otherwise;
- d^{exit}_{nt} = distance to off-ramp driver intends to use;
- Δexit^i = number of lane changes required to get from lane *i* to the exit lane;
- $\delta^{\text{next exit}}_{nt}$ = indicator with value 1 if the driver intends to take the next exit, and 0 otherwise;
- $\delta^{i,\Delta\text{exit}\geq 1}_{nt}$ = indicator with value 1 if lane *i* is not the exit lane, and 0 otherwise; and
- $\delta^{i,i}_{n,t-1}$ = state indicator with value 1 if lane *i* was the target lane in time *t*–1.

Gap Acceptance Model

The estimated lead and lag gaps are given by

$$\begin{aligned} \ln\left(\text{CG}^{\text{lead}}_{nt}\right) &= 1.706 - 6.323\max\left(0, \Delta V^{\text{lead}}_{nt}\right) - 0.155\min\left(0, \Delta V^{\text{lead}}_{nt}\right) \\ &\quad + 0.099\upsilon_{n} + \epsilon^{\text{lead}}_{nt} \\ &\quad \sim N\left(0, 0.939^{2}\right) \end{aligned} \tag{14}$$

where $\Delta V^{\text{lead}}_{nt}$ and $\Delta V^{\text{lag}}_{nt}$ are the relative lead and lag speeds, respectively. They are defined as the speed of the lead (or lag) vehicles less the subject speed.

$$\begin{aligned} \ln\left(\text{CG}^{\text{lag}}_{nt}\right) &= 1.429 + 0.512\max\left(0, \Delta V^{\text{lag}}_{nt}\right) + 0.211\upsilon_{n} + \epsilon^{\text{lag}}_{nt} \\ &\quad \sim N\left(0, 0.775^{2}\right) \end{aligned} \tag{15}$$

The lead critical gap decreases with the relative lead speed, that is, it is larger when the subject is faster relative to the lead vehicle. The effect of the relative speed is strongest when the lead vehicle is faster than the subject. In this case, the lead critical gap quickly reduces to almost zero as the relative speed is increasingly positive. This result suggests that drivers perceive very little risk from the lead vehicle when it is getting away from them.

Inversely, the lag critical gap increases with the relative lag speed: the faster the lag vehicle is relative to the subject, the larger is the lag critical gap. In contrast to the lead critical gap, the lag gap does not diminish when the subject is faster, but keeps a minimum critical gap.

Estimated coefficients of the unobserved driver characteristics variable are positive for lead and lag critical gaps. This result is consistent with the interpretation of this variable as positively related to timid drivers that require larger gaps for lane changing compared with more aggressive drivers.

TABLE 2 Likelihood Values of Estimated Models

	Model	Likelihood Value	Parameters
1	No state dependence model	−880.35	25 (23)
2	State dependence model with initial values for all the variables	−874.97	38 (24)
3	State dependence model	−876.19	29 (24)

Model Selection

To evaluate the importance in regard to model fit of the integration of the HMM structure in the lane changing model, the proposed model was compared with a model with similar structure and explanatory variables that does not account for state dependency. The maximum likelihood values and numbers of parameters of three different models (the model with no state dependence and two state dependence models, with the full set of initial conditions parameters and the final model presented above) are shown in Table 2. The numbers of parameters in parentheses are only those that are actually used for prediction (excluding the parameters for the initial conditions model and those that capture the impact of the unobserved downstream path plan).

Likelihood ratio tests were applied to select among these models. The test statistic value for the comparison between Models 1 and 2 is 10.76 with 13 degrees of freedom ($p = .63$). Therefore, Model 1 cannot be rejected. The test statistics for Models 1 and 3 are 8.32 with four degrees of freedom ($p = .08$), and so Model 3 may be preferred.

CONCLUSION

This paper presented a lane-changing model that accounts for state dependence in the underlying target lane choices drivers make over time. The model is based on integration of an HMM within the structure of the lane-changing model. The resulting model accounts for heterogeneity in the driver population by introducing an individual-specific error term and for dependence of target lane choices drivers make on previous ones. The model includes two choice components: the selection of a target lane and gap acceptance. A random utility approach is adopted to model both choices.

The inclusion of state dependence in the model creates conditionality on the initial target lane the driver chose. However, the initial conditions are not observed and depend on the individual-specific error term. Ignoring this endogeneity results in inconsistent parameter estimates. A method to overcome this problem is proposed and applied to estimate the model parameters using detailed trajectory data. The estimation results indicate that the impacts of heterogeneity and state dependence are significant in lane-changing behavior. Statistical tests for model selection recommend the state dependence model over a model that ignores state dependence.

Although the results presented in this paper are promising, additional tests with more data sets are needed to validate the usefulness of incorporating state dependence in lane-changing models. The HMM frame presented could also be incorporated in multilevel lane-changing models [e.g., Hidas (*10*) and Toledo et al. (*11*)] that can better represent behavior in heavily congested traffic. Furthermore, the model presented here assumed Markovian state dependency. Other forms of dependency, such as semi-Markovian models, in which the time a driver has been in a specific state (targeting the same lane) is modeled, may be more appealing. Finally, the impact on the emergent macroscopic traffic flow characteristics also needs to be studied through their implementation in microscopic simulation models.

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