Application of Cross-Nested Logit Route Choice Model in Stochastic User Equilibrium Traffic Assignment

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Most stochastic user equilibrium (SUE) model applications reported in the literature are based on the multinomial logit (MNL) model. This paper presents a SUE assignment based on the cross-nested logit (CNL) route choice model, which can better represent route choice behavior. The paper develops path-based algorithms to solve the CNL-SUE problem based on adaptation of the disaggregate simplicial decomposition method. The algorithms differ for the step-size determination; three different methods are considered. The algorithms are tested in two well-known networks. Two main tests are conducted: (*a*) the impact of the CNL model parameters on the assignment results is analyzed and (*b*) the differences between the CNL-SUE and the MNL-SUE solutions are investigated. The results indicate that the path-based algorithm with Armijo's step-size rule outperforms other step-size determinations. This paper indicates that depending on the model parameters, particularly the nesting coefficient, the CNL-SUE path flows may be quite different from the MNL-SUE path flows.

The deterministic static traffic assignment model assumes that drivers have complete and accurate information on the state of the network when they make their route choices and so are able to select optimal routes. Stochastic user equilibrium (SUE) traffic assignment, which is defined as a state in which drivers cannot improve their perceived travel times by unilaterally changing routes (*1*), relaxes this assumption. Instead, the SUE model assumes that the traffic assignment follows a probabilistic route choice model. Most of the applications of SUE models reported in the literature are based on the multinomial logit (MNL) model. Special properties of the MNL model make it possible to use efficient solution algorithms that avoid explicit enumeration of the route choice set (*2, 3*) to solve the MNL-SUE problem. However, the MNL model assumes that the utilities of different routes, even overlapping ones, are uncorrelated. This may lead to counterintuitive assignment results (*4–6*).

The MNL model exhibits the property of independence of irrelevant alternatives (IIA), which turns out to be a deficiency of the MNL model in the route choice context and can be interpreted as a failure to account for similarities between alternatives. Daganzo and Sheffi (*1*) presented simple network examples to illustrate counterintuitive results that may be obtained when the MNL model is applied for route choice. Instead, they proposed using the multinomial probit (MNP) model, which can account for similarity between alternatives. However, the MNP model is unattractive from a computational standpoint because the probability function cannot be expressed in closed form.

In recent years, a number of other discrete choice model structures were adapted to route choice behavior in an attempt to overcome the limitation of the MNL model and capture the impact of similarity among various routes on drivers' perceptions and decisions. These models range from modifications of MNL, such as C-logit (*7*) and path-size logit (*8, 9*) that capture similarities through additional terms in the systematic utilities of the various routes to models based on the generalized extreme value theory, such as paired combinatorial logit (PCL) (*10, 11*) and cross-nested logit (CNL) (*10, 12*), and logit kernel models (*9, 13*), which capture these similarities by allowing more general correlation structures. Although formulations of the SUE assignment problem that correspond to some of these route choice models have been introduced in the literature (*14*), there has been little study of the practical implications of using them on the assignment results and even less development of appropriate algorithms for their solution.

This paper presents a SUE assignment based on the CNL route choice model. Algorithms for solution of the CNL-SUE problem are developed and tested; they are based on adaptation of the disaggregate simplicial decomposition (DSD) method developed by Damberg et al. (*15*) for the MNL-SUE problem. These algorithms are used to solve large-scale traffic networks and to test the impact of parameters of the route choice models on the assignment results and on the difference between the CNL-SUE and MNL-SUE solutions.

The rest of this paper is organized as follows. First, a brief review of the CNL route choice model is presented, followed by formulation of the CNL-SUE assignment problem. The adaptation of the DSD algorithm to the CNL-SUE problem is presented next. Then case studies are presented that evaluate the performance of the proposed algorithm and compare the assignment results and computational effort with those of the MNL-SUE problem using two well-known test networks. The final section summarizes the findings and discusses the potential use of the different route choice models in traffic assignment problems.

CNL ROUTE CHOICE MODEL

The CNL model was adapted to route choice situation by Prashker and Bekhor (*10*) and Vovsha and Bekhor (*12*). Their adaptation uses a two-level nesting structure in which the upper level (nests) includes all the links in the network. The lower level consists of all the routes in the set C of routes that connect between the origin and destination. Each of the routes is allocated to all the nests that represent links that

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Transportation Research Record: Journal of the Transportation Research Board, No. 2003, Transportation Research Board of the National Academies, Washington, D.C., 2007, pp. 41–49.
DOI: 10.3141/2003-06

are part of it. Assuming this structure, the probability (P) of choosing route k is given by

$$P(k)=\frac{\exp\left[-\theta c_k+\ln\sum_{m\in M}\alpha_{mk}\left(\sum_l \alpha_{ml}\exp(-\theta c_l)\right)^{\mu-1}\right]}{\sum_{j\in C}\exp\left[-\theta c_j+\ln\sum_{m\in M}\alpha_{mj}\left(\sum_l \alpha_{ml}\exp(-\theta c_l)\right)^{\mu-1}\right]} \tag{1}$$

where

j, k, l = route indicators;
m = nest indicator;
M = set of links that compose path k;
c_k = generalized cost of travel on path k;
θ = dispersion parameter that determines the sensitivity of route choices to changes in travel costs;
μ = parameter that indicates the degree of nesting, as in the nested logit model: when $\mu = 1$, the model collapses to MNL, and when $\mu \to 0$, the model becomes probabilistic at the higher (link) level and deterministic at the lower (path) level; and
α_{mk} = parameters that determine allocation of route k among the links that compose m.

It is possible to rewrite the expression for the probability of choosing a route as follows:

$$P(k)=\sum_m P(m)P(k|m) \tag{2}$$

where the conditional probability of a route k being chosen in link (nest) m is

$$P(k|m)=\frac{\left(\alpha_{mk}\exp(-\theta c_k)\right)^{1/\mu}}{\sum_l\left(\alpha_{ml}\exp(-\theta c_l)\right)^{1/\mu}} \tag{3}$$

and the marginal probability of a nest m being chosen is

$$P(m)=\frac{\left(\sum_k\left(\alpha_{mk}\exp(-\theta c_k)\right)^{1/\mu}\right)^{\mu}}{\sum_b\left(\sum_k\left(\alpha_{bk}\exp(-\theta c_k)\right)^{1/\mu}\right)^{\mu}} \tag{4}$$

where b is a nest indicator.

The potentially large number of allocation parameters poses a difficulty in calibrating the model. Prashker and Bekhor (*10*) proposed determining these parameters exclusively based on the network topology using the physical length of the links that are common to various routes with the following function:

$$\alpha_{mk}=\left(\frac{L_m}{L_k}\right)^{\gamma}\delta_{mk} \tag{5}$$

where

L_m = length of link m,
L_k = length of route k,
δ_{mk} = 1 if link m is on route k and 0 otherwise, and
γ = parameter that reflects drivers' perception of similarity among routes.

FORMULATION OF THE CNL-SUE ASSIGNMENT PROBLEM

Bekhor and Prashker (*14*) presented mathematical program formulation, which corresponds to the CNL-SUE assignment problem—that is, the CNL model is derived as the first-order conditions for the solution of this program. This formulation is similar to the one derived by Fisk (*16*) for MNL-SUE. The CNL-SUE assignment is obtained as the solution of the following mathematical program:

$$\min\ Z = Z_1+Z_2+Z_3$$

$$Z_1=\sum_a\int_0^{x_a} c_a(w)dw$$

$$Z_2=\frac{\mu}{\theta}\sum_{rs}\sum_m\sum_k f_{mk}^{rs}\ln\frac{f_{mk}^{rs}}{\left(\alpha_{mk}^{rs}\right)^{1/\mu}}$$

$$Z_3=\frac{1-\mu}{\theta}\sum_{rs}\sum_m\left(\sum_k f_{mk}^{rs}\right)\ln\left(\sum_k f_{mk}^{rs}\right)$$

$$\text{s.t.}\ \sum_m\sum_k f_{mk}^{rs}=q^{rs}\qquad\forall\ r,s$$

$$f_{mk}^{rs}\ge 0\qquad\forall\ m,k,r,s \tag{6}$$

where

x_a and c_a = flow and cost, respectively, on link a;
w = flow variable;
f_{mk}^{rs} = flow on path k of nest m between origin r and destination s; and
q^{rs} = demand for travel from r to s.

The expression

$$f_{mk}^{rs}\ln\frac{f_{mk}^{rs}}{\left(\alpha_{mk}^{rs}\right)^{1/\mu}}$$

is defined as zero if either $f_{mk}^{rs}=0$ or $\alpha_{mk}^{rs}=0$.

The preceding formulation is composed of three terms. The first term (Z_1) is identical to the deterministic user equilibrium formulation. The second term (Z_2) is an entropy term similar to Fisk's formulation for the MNL-SUE problem but modified to include the allocation coefficients and the nesting coefficient. The third term (Z_3) is also an entropy term, where the flows f_{mk}^{rs} are aggregated by all routes. Bekhor and Prashker (*14*) showed that the entropy terms (Z_2) and (Z_3), respectively, correspond to the conditional and marginal probabilities of choosing a route. The constraints of the problem are similar to other mathematical formulations for the equilibrium problem (conservation equations and nonnegativity of path flows).

There are several important modifications in the preceding mathematical program compared with Fisk's formulation for the MNL-SUE problem. The problem itself is defined in terms of f_{mk}^{rs}, a decomposition of path flows to the m links. In addition, the entropy term (Z_2) is modified to include the allocation coefficients. Finally, a second entropy term (Z_3), which corresponds to the higher choice level, is also added.

Assuming that the link cost functions are continuous monotonically increasing functions of link flows, Bekhor and Prashker (*14*) showed that the objective function is continuous and convex. For finite demand, its derivatives exist and are bounded, and therefore it is also Lipschitz continuous. In addition, if the nesting coefficient is equal to 1, the preceding formulation collapses to the SUE-MNL formulation.

Few studies have actually implemented the CNL-SUE. Bekhor and Prashker (*17*) applied their formulation using the method of successive averages (MSA) algorithm (*18*) to a small network. Chen et al. (*19*) developed an algorithm based on the partial linearization method for solving the PCL-SUE problem. The PCL-SUE formulation is similar to Equation 6, as demonstrated by Bekhor and Prashker (*14*). In this paper, the DSD algorithm proposed for the MNL-SUE problem by Damberg et al. (*15*) is adapted to the CNL-SUE problem.

DSD ALGORITHM FOR CNL-SUE PROBLEM

Damberg et al. (*15*) extended the DSD algorithm of Larsson and Patriksson (*20*) to solve the MNL-SUE problem. The method is based on iterative solution of subproblems that are generated through partial linearization of the objective function. The new iteration solution is found as a convex combination of the solution of the linearized subproblem and the previous iteration solution. This section presents the adaptation of the method to the CNL-SUE problem.

Suppose that at iteration n a feasible path-flow solution is given. The first term in Formulation 6 is linearized, which amounts to assuming that travel costs are fixed at their current values. The resulting subproblem is given by

$$\min Z = \sum_{rs}\sum_{k} c_k^{rs(n)} f_k^{rs} + \frac{\mu}{\theta}\sum_{rs}\sum_{m}\sum_{k} f_{mk}^{rs} \ln \frac{f_{mk}^{rs}}{\left(\alpha_{mk}^{rs}\right)^{1/\mu}} + \frac{1-\mu}{\theta}\sum_{rs}\sum_{m}\left(\sum_{k} f_{mk}^{rs}\right)\ln\left(\sum_{k} f_{mk}^{rs}\right) \quad (7)$$

where $c_k^{rs(n)}$ is the travel cost on path k based on the vector of path flows at iteration n. The solution to this subproblem is given by the CNL model route choices

$$h_k^{rs(n)} = q^{rs}\frac{\exp\left[-\theta c_k^{rs(n)} + \ln\sum_{m}\alpha_{mk}^{rs}\left(\sum_{l}\alpha_{ml}^{rs}\exp\left(-\theta c_k^{rs(n)}\right)\right)^{\mu-1}\right]}{\sum_{j}\exp\left[-\theta c_j^{rs(n)} + \ln\sum_{m}\alpha_{mj}^{rs}\left(\sum_{l}\alpha_{ml}^{rs}\exp\left(-\theta c_l^{rs(n)}\right)\right)^{\mu-1}\right]} \quad (8)$$

If the vector $h^{(n)} - f^{(n)}$ is nonzero, it defines a descent direction with respect to the objective Function 6—that is, a new solution that would be generated by taking an appropriate step size in this direction would decrease the value of the objective function. The new solution is given by

$$f_k^{rs(n+1)} = f_k^{rs(n)} + \lambda^{(n)}\left(h_k^{rs(n)} - f_k^{rs(n)}\right) \qquad \forall k, \forall rs \quad (9)$$

where $\lambda^{(n)}$ is the step size in iteration n. In this paper, three different methods to calculate the step size are considered: an exact calculation of the optimal step size, approximation of the optimal step size using Armijo's rule, and application of predetermined step sizes as in the MSA algorithm.

An exact optimal step size is calculated by using the golden section line search method as the optimal solution of the following problem:

$$\lambda^{(n)} = \arg\min_{\lambda\in[0,1]} Z\left[f^{(n)} + \lambda\left(h^{(n)} - f^{(n)}\right)\right] \quad (10)$$

The exact line search may be computationally expensive to perform in the case of the CNL-SUE problem, because the variable of interest is f_{mk}^{rs} (the flow on path k of nest m between r and s). The dimension of this variable may be very large even for moderately sized networks. Consequently, the number of operations required to calculate the objective function value and the overall effort to find the optimal step size may be very large. An alternative approach is to use Armijo's approximate step-size rule (*21*), which is defined as follows:

$$\lambda^{(n)} = \beta^{m_k} \quad (11)$$

where m_k is the first integer, $m \geq 0$, which satisfies

$$Z\left(f^{(n)}\right) - Z\left(f^{(n)} + \beta^m\left(h^{(n)} - f^{(n)}\right)\right) \geq -\epsilon\beta^m \nabla Z\left(f^{(n)}\right)\left(h^{(n)} - f^{(n)}\right) \quad (12)$$

and $0 < \beta < 1$ and $0 < \epsilon < 1$ are parameters.

Finally, the simplest approach to the step-size calculation is use of predetermined step sizes. These approaches require very little effort in each iteration but may require many more iterations to reach convergence. The following step-size rule, which simplifies the proposed algorithm to a path-based MSA method, was applied:

$$\lambda^{(n)} = \frac{1}{n+1} \quad (13)$$

The flows calculated in Equation 9 are then used to update link costs and path costs. A new subproblem with the updated path costs is solved by using Equation 8 to produce a new descent direction. This iterative process continues until the convergence criterion is satisfied.

CASE STUDIES

The three variations of the algorithm (i.e., with MSA, Armijo, and golden section step sizes) were implemented and tested on two well-known networks: Sioux Falls, South Dakota (*22*), and Winnipeg, Manitoba, Canada (*23*). Two tests were conducted to evaluate the following questions.

Test 1 evaluates the performance of the proposed algorithm for solution of the CNL-SUE and compares the relative efficiency of the three step-size calculation methods in terms of computational effort. The three methods require increasing amounts of work per iteration. However, the more complex methods may require a smaller number of iterations to converge to the optimal solution. The performance of the algorithms may depend on the parameters of the problem; therefore, the calculation was repeated for the two networks for different values of the dispersion parameter θ and the nesting coefficient μ.

Test 2 compares the CNL-SUE assignment results and required computational effort with those obtained for the MNL-SUE problem. The CNL-SUE problem formulation uses decomposed path-link flows f_{mk}^{rs} as decision variables and not directly the path flows f_k^{rs} as in MNL-SUE. This significantly increases the dimension of the problem. As a result, the effort required to solve the problem is also larger. However, the CNL route choice model is a more realistic representation of drivers' behavior. The aim of this test is to quantify the trade-off between behavioral realism and computational effort with the two assignment models.

Path-based assignment requires generating a choice set of travel routes. In this study, predetermined route sets are used that are not augmented during the various runs. Although column generation methods may be used to update the route set during the assignment [see Damberg et al. (*15*) for a discussion], this approach provides a common basis for comparison of all algorithms and assignment models. Routes were generated by using a combination of the link elimination method of Azevedo et al. (*24*) and the penalty method of de la Barra et al. (*25*) with a penalty of 5% on travel times on the shortest route links. Only acyclic routes were considered in these methods.

The Sioux Falls network (Figure 1) is composed of 24 nodes, 76 links, and 550 origin–destination (O-D) pairs. The choice set generation method created an average of 6.3 routes for each O-D pair. The maximum number of routes generated for any O-D pair was 13. The Winnipeg network (Figure 2) includes 948 nodes, of which 154 are zone centroids, 2,535 are links, and 4,345 are O-D pairs with positive demand for travel. The total demand is 54,459 trips; 174,491 unique routes were generated (an average of 40.1 routes per O-D pair). The maximum number of routes generated for any O-D pair was 50. The thick black lines in Figure 2 indicate the links that belong to the path set of a specific O-D pair. The Bureau of Public Roads function with link-specific parameters is used in both networks.

RESULTS

Test 1. Algorithm Performance

The various algorithms were run on a PC with a 3.0-GHz Pentium 4 processor. Figures 3 and 4 present four graphs each showing the number of iterations and the central processing unit (CPU) time, respectively, needed to reach convergence in the Sioux Falls network as a function of the value of the parameter θ for various values of μ.

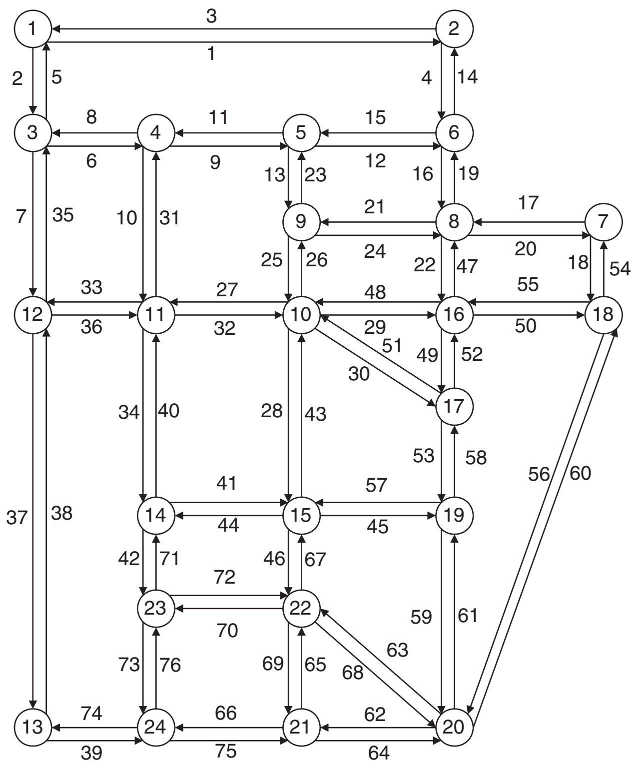


FIGURE 1 Sioux Falls network (*26*).

FIGURE 2 Winnipeg network (*23*).

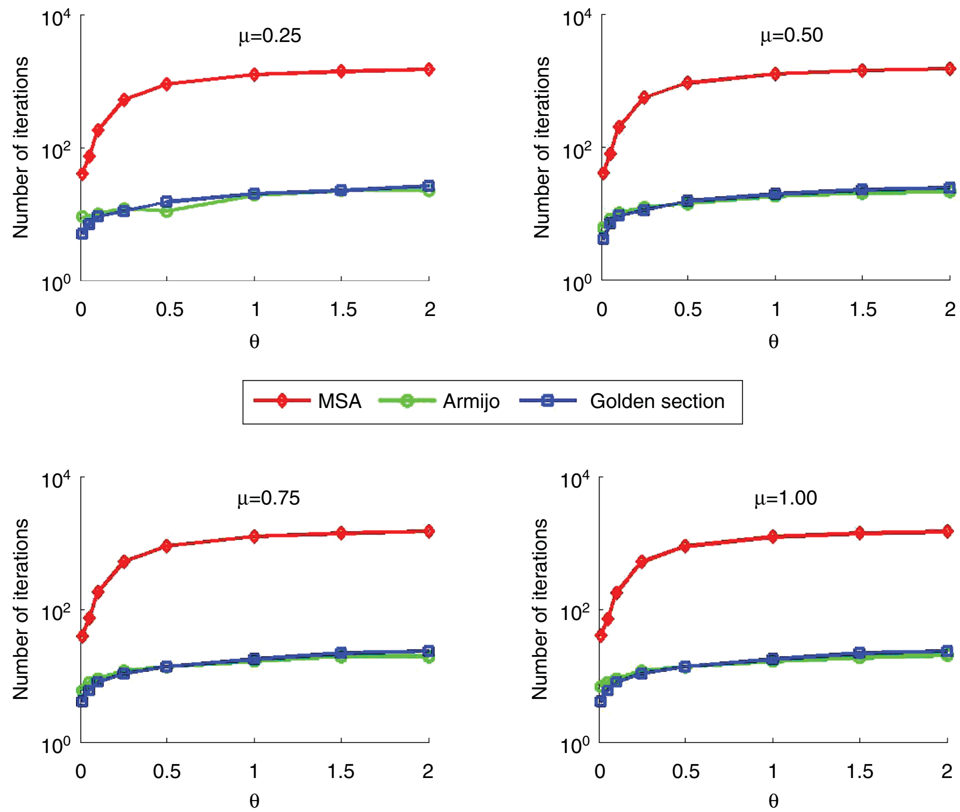


FIGURE 3 Iterations required to solve Sioux Falls network.

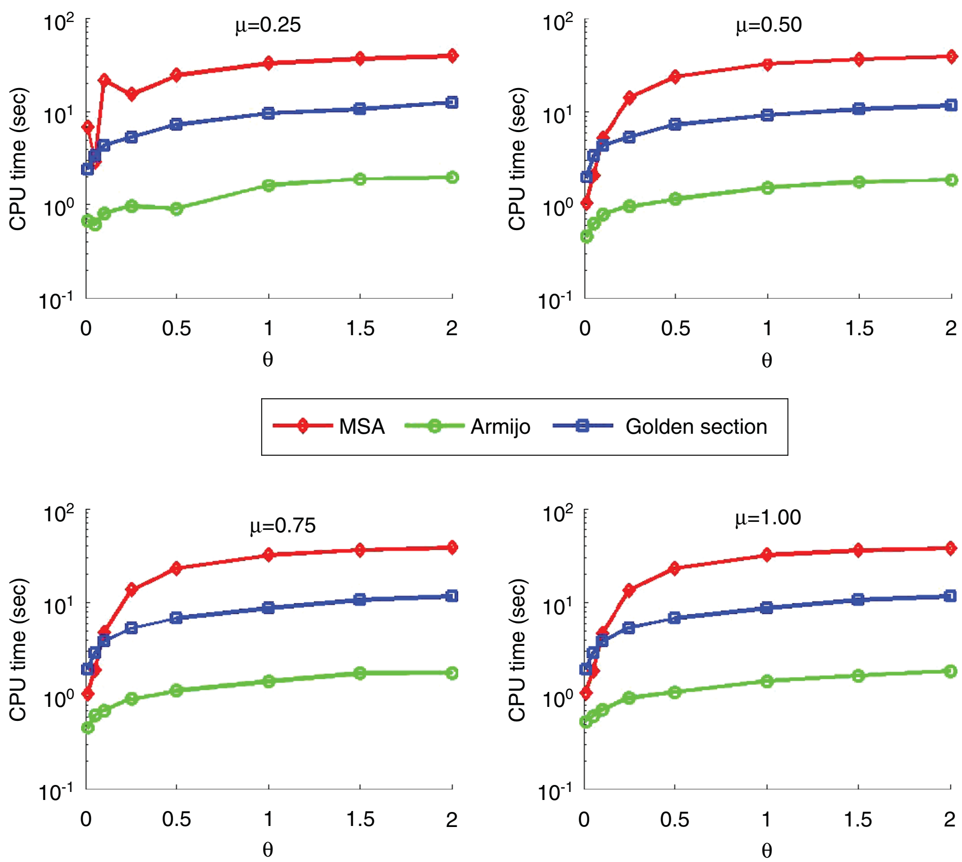


FIGURE 4 CPU times required to solve Sioux Falls network.

The y axes in Figures 3 and 4 have logarithmic scales. Convergence was measured by the root-mean-square error (RMSE) of the internal inconsistency of the solution:

$$\text{RMSE}^{(n)} = \sqrt{\frac{1}{K}\sum_{rs}\sum_{k}\left(h_k^{rs(n)} - f_k^{rs(n)}\right)^2} \quad (14)$$

where K is the total number of routes in the choice sets. Figures 3 and 4 are based on the criterion $\text{RMSE}^{(n)} \leq 0.0001$.

When the dispersion parameter θ is smaller than 0.5, fewer iterations are required to obtain convergence. For values of θ that are higher than 0.5, the level of effort required to converge, in terms of the number of iterations and CPU time, is similar in all cases. In the problem formulation in Equation 6, the terms Z_2 and Z_3 depend on the reciprocal of this parameter. When the value of θ is small, the contribution of these two terms to the overall objective function is very large and dominates that of the term Z_1. Their contribution then decreases as θ increases. The solution algorithm is based on a linear approximation of the term Z_1 in each iteration, and so the descent direction is more accurate when θ is small. This result is consistent with findings discussed by Prashker and Bekhor (*27*). For larger values of θ, Z_1 dominates the objective function and the problem becomes insensitive to changes in the value of θ. The nesting parameter μ appears to have little impact on the performance of the algorithms.

The variants that use the step sizes based on the Armijo rule and the golden section search exhibit similar performance in terms of numbers of iterations. Both largely outperform the MSA algorithm in all cases. However, the amount of work required by the algorithms in each iteration differs. The golden section search requires the most work per iteration, and the MSA requires the least. As a result, for small values of θ, in which all algorithms require relatively small numbers of iterations to converge, the MSA outperforms the golden search in terms of CPU time. As θ increases, more iterations are required, in particular by MSA, and its performance deteriorates. For the Sioux Falls network, it uses more CPU time than the golden section search for θ ≥ 0.1. However, the Armijo rule provides the best convergence results in all cases. The amount of work it requires per iteration is significantly smaller than for the golden section search. The reason for this is that it performs fewer evaluations of the objective function in Equation 6, which is time-consuming because of the summations over the very large number of link path flows f_{mk}^{rs}. Still, the Armijo rule requires roughly the same number of iterations to converge as the golden section search. It requires three to four times more work per iteration than MSA, but it takes one to two orders of magnitudes fewer iterations to converge. Therefore, it outperforms both the MSA and the golden section search.

The performance results for the Winnipeg network are similar to those for Sioux Falls and, for brevity, are not presented in this paper. The Armijo rule and the golden section search take similar numbers of iterations to converge. But, in terms of CPU time, the Armijo rule is superior to the other methods. The impact of the value θ on the performance is also similar to the Sioux Falls network, with θ ≈ 0.5 marking the point beyond which the performance is insensitive to changes in the value of θ.

Test 2. Comparison with MNL-SUE Assignment

Figures 5 and 6 show the difference between the CNL-SUE and MNL-SUE assignment results as a function of the value of the problem parameters (θ and μ) for the Sioux Falls and Winnipeg networks, respectively. The difference between the two assignment solutions was measured in each case by the RMSE:

$$\text{RMSE} = \sqrt{\frac{1}{K}\sum_{rs}\sum_{k}\left(f_{k,\text{MNL}}^{rs*} - f_{k,\text{CNL}}^{rs*}\right)^2} \quad (15)$$

where $f_{k,\text{MNL}}^{rs*}$ and $f_{k,\text{CNL}}^{rs*}$ are the convergence path flows in the MNL-SUE and CNL-SUE assignment, respectively.

The results indicate that the differences between the two solutions may be substantial. To put the RMSE statistics in context, it may be noted that the average path flow is 0.11 in the Sioux Falls network and 0.31 in the Winnipeg network. The nesting parameter μ, which determines the degree of correlation among overlapping routes, has a very strong impact on this difference, which decreases when μ increases. This result is also consistent with the CNL model theory, which shows that, when this parameter tends to 1, the CNL results become closer to the MNL results, as indicated by the diminishing RMSE.

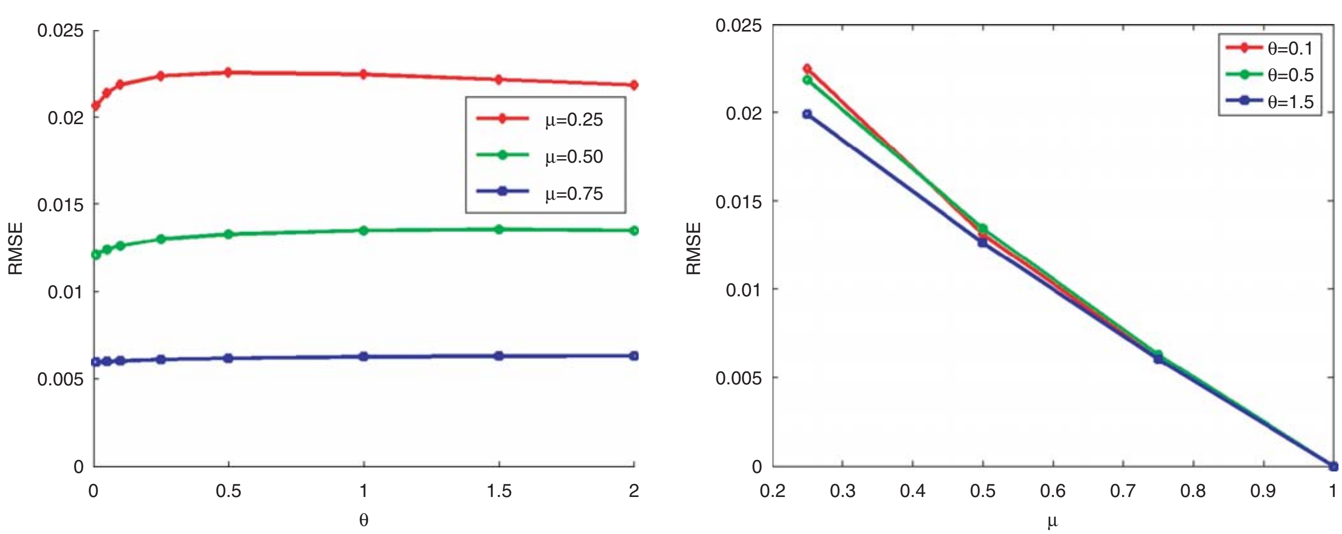


FIGURE 5 Impact of parameters θ and μ on difference between CNL-SUE and MNL-SUE solutions in Sioux Falls network.

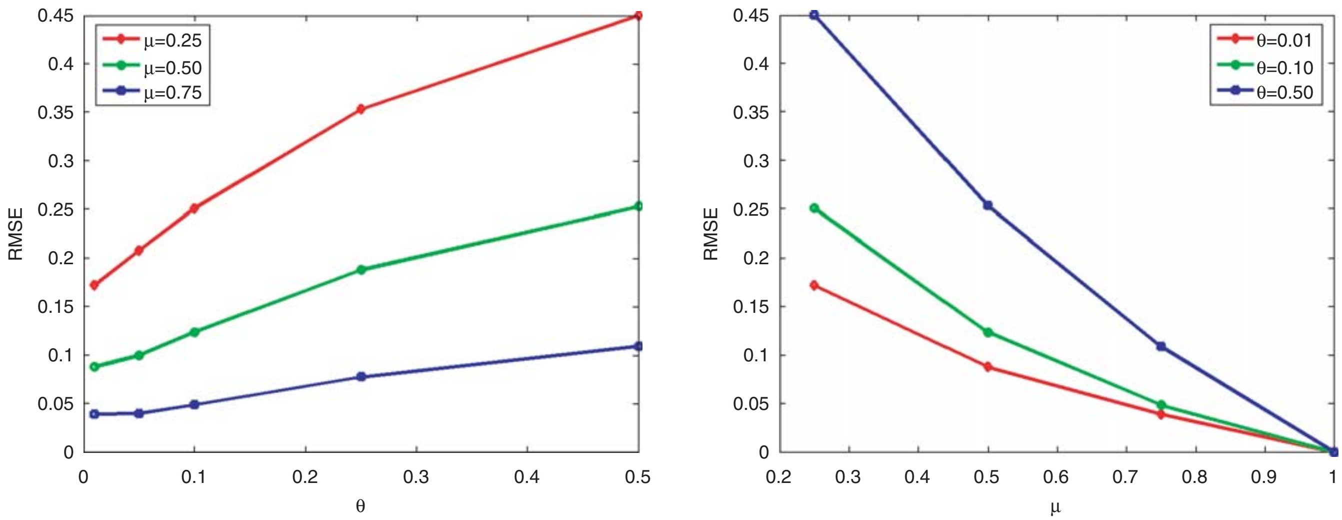


FIGURE 6 Impact of parameters θ and μ on difference between CNL-SUE and MNL-SUE solutions in Winnipeg network.

Different results are observed in the two networks with respect to the dispersion parameter θ. Whereas this parameter does not affect the results in the Sioux Falls network (RMSE slightly decreases with increasing values of θ), the RMSE increases with increasing values of θ in the Winnipeg network. The explanation in this case is related to the range of the parameter combined with the congestion level in the networks. Recall that when θ tends to zero, the path flows tend to be distributed equally among the routes, meaning that MNL-SUE and CNL-SUE flows will tend to be equal. On the other hand, when θ tends to infinity, the SUE problem collapses to the deterministic assignment problem, regardless of the choice model, and again MNL-SUE and CNL-SUE flows will tend to be equal. This means that a curved-shape behavior for the difference between CNL-SUE and MNL-SUE flows is expected as a function of the value of θ.

In the Sioux Falls network, congestion is higher than in the Winnipeg network. This means that congestion term (Z_1) in the objective function, which is independent of θ, is more dominant in this network. Therefore, the impact of θ is small in this network. In the Winnipeg network, which is relatively less congested, the parameter range considered (between 0.01 and 0.5) significantly affects the flow difference between the two models.

To illustrate the results at the disaggregate level, a comparison between path flows obtained from each model for a specific O-D pair in the Winnipeg network (displayed in Figure 2). Figure 7 shows the

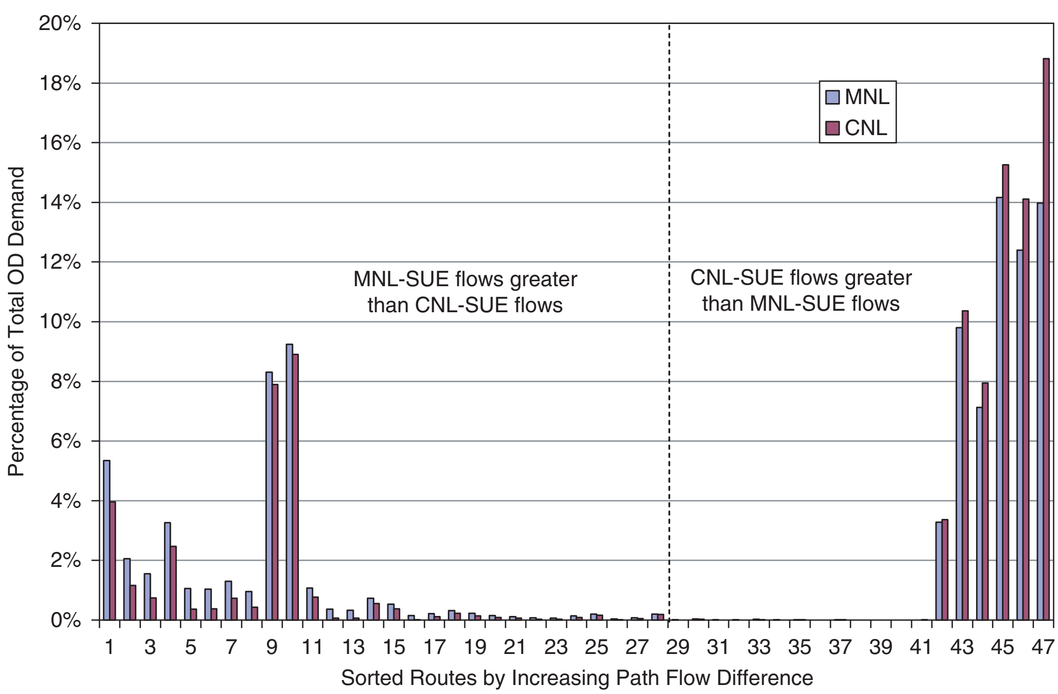
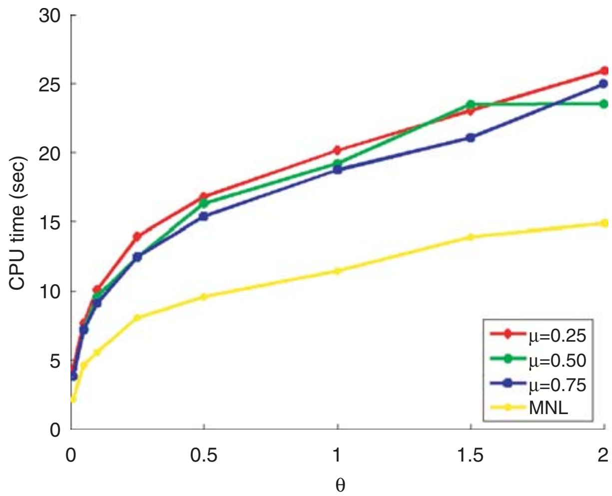


FIGURE 7 Path flow comparison for a specific O-D pair in Winnipeg network.



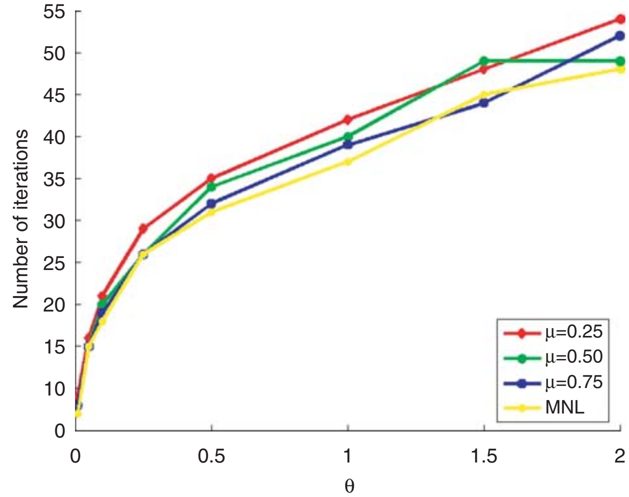


FIGURE 8 Algorithm performance of MNL-SUE and CNL-SUE assignment in Sioux Falls network.

path flow difference for this specific O-D pair, obtained with $\theta = 0.5$ for both models and $\mu = 0.5$ for the CNL model. The routes were sorted by increasing order of the difference between CNL-SUE and MNL-SUE path flows.

The figure shows that the amount of flow in most of the 47 paths is quite small. The 10 highest path flows carry 87% and 93% of the total demand for the MNL-SUE and CNL-SUE, respectively. However, the path flows in these routes can be quite different. For example, the highest MNL-SUE path flow carries 14% of the total demand, and the corresponding highest CNL-SUE path flow is about 19% of the total demand, or 35% higher.

An additional test was performed to compare the computational performance when using the two route choice models. Figure 8 shows the number of iterations and the CPU time required to solve the MNL-SUE model and CNL-SUE model as a function of the problem parameters (θ and μ) for the Sioux Falls network. The Armijo rule was used in all assignment runs.

The encouraging result is that, whereas in most cases MNL-SUE can be solved faster, the differences are not prohibitively large in most cases. The computational effort increases with increasing values of the dispersion parameter θ. The computational effort also increases when the nesting parameter μ decreases (i.e., when the correlations among overlapping routes are higher). Higher correlations indirectly introduce impacts of flows on one route on other routes and so may slow convergence.

In addition to the results presented in Figure 8, Table 1 summarizes selected results for both networks tested, assuming $\theta = 0.5$ for both models and $\mu = 0.5$ for the CNL model. Table 1 shows that the Armijo rule for step-size determination performs quite well for both networks presented, but more tests are needed to verify the efficacy of the rule.

SUMMARY AND CONCLUSIONS

This paper discussed path-based algorithms to solve the CNL-SUE problem. The aim of the paper was to incorporate more realistic route choice models into the traffic assignment problem. The results of the paper show that it is possible to implement such algorithms with affordable computer resources. In particular, the Armijo step-size rule was found to perform well for the networks tested, but more experiments need to be conducted to verify whether the rule also works well for extended problems.

The CNL route choice model can overcome deficiencies of the MNL model. However, in heavy congested networks, both models

TABLE 1 Summary of Algorithm Results (θ = 0.5)

Network	Route Choice Model	Step Size Method	Total Number of Paths	Objective Function Value	Iterations to Converge	CPU Time (s)	Time per Iteration (s)
Sioux Falls	MNL	MSA	3,441	–269.29	147	1.7	0.012
Sioux Falls	MNL	Armijo	3,441	–269.29	31	1.4	0.045
Sioux Falls	MNL	G-S	3,441	–269.29	29	8.7	0.300
Sioux Falls	CNL	MSA	3,441	–229.75	144	3.6	0.025
Sioux Falls	CNL	Armijo	3,441	–229.75	32	2.8	0.086
Sioux Falls	CNL	G-S	3,441	–229.75	22	10.2	0.463
Winnipeg	MNL	MSA	174,491	917,976	>10E5	>10E6	77
Winnipeg	MNL	Armijo	174,491	917,976	28	9,967	356
Winnipeg	MNL	G-S	174,491	917,976	26	55,510	2,135
Winnipeg	CNL	MSA	174,491	989,445	>10E5	>10E6	128
Winnipeg	CNL	Armijo	174,491	989,445	50	21,597	432
Winnipeg	CNL	G-S	174,491	989,445	46	102,120	2,220

result in similar flows. The differences are more pronounced in moderately congested networks, as in the Winnipeg network. This means that the CNL-SUE model can better represent traffic phenomena for any congestion level than the MNL-SUE model because of the advantages of the CNL over the MNL model. This is also true for other equilibrium procedures such as MNP-SUE and PCL-SUE, and further research will address the trade-offs (specifically, algorithm performance versus path-flow difference) between these models and the CNL-SUE model.

The tests presented in this paper focused on two model parameters: the dispersion parameter and the nesting coefficient. The demand matrices for the two networks in all tests were fixed. As suggested in this paper, there is a need to investigate further the effect of different demand levels on the algorithm performance and to verify the difference between CNL-SUE and MNL-SUE flow patterns.

In this paper, the route choice model is a function of travel times only. The formulation of the problem can accommodate additional explanatory variables, similar to the "generalized cost" variable in deterministic traffic assignment problems. The generalized cost is a linear transformation of the travel time and cost. More complex utility functions are yet to be implemented in traffic assignment models.

The results presented in this paper are based on several assumptions common to simple equilibrium models: static assignment, fixed demand, separable volume-delay function, single-user class. Additional research is needed to extend and verify the CNL-SUE model for more general problems.

The results presented in this paper were obtained for a fixed set of routes. The results may be different for other choice set compositions and sizes. A recent paper by Bekhor et al. (*28*) compares MNL-SUE and CNL-SUE models for different path-set sizes generated before the assignment process, similar to this paper. The effect of column generation methods in the algorithm performance and path-flow results is being investigated, and the results will be presented in a future paper.

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The Transportation Demand Forecasting Committee sponsored publication of this paper.