# Stochastic User Equilibrium for Route Choice Model Based on Random Regret Minimization

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A static stochastic user equilibrium (SUE) problem was formulated: the mode of random regret minimization (RRM) was used for route choices. The RRM approach assumes that individuals minimize anticipated regret, rather than maximize expected utility, when choosing from alternative routes. The cost function for the RRM model is not separable, and so a variational inequality approach was adopted to formulate the problem. A path-based algorithm was applied to solve the RRM-SUE problem with the method of successive averages. Implementation of the algorithm in a real-world network is illustrated, and the trade-offs and differences between the proposed model and the SUE based on random utility models is discussed.

The random regret minimization (RRM) model (1) is an alternative to random utility maximization (RUM) models of travel choice. The RRM approach assumes that individuals minimize anticipated regret, rather than maximize expected utility, when choosing routes. Regret occurs when one or more nonchosen alternatives perform better than a chosen alternative for one or more attributes. The RRM model features multinomial nested logit (MNL) choice probabilities and can be estimated with conventional discrete choice software packages. In a number of recent empirical studies, the RRM paradigm (particularly its MNL model form) has been shown to provide a useful representation of behavior in several travel decision-making contexts, including route, departure time, destination, parking lot, travel information acquisition, and vehicle purchase choices (2, 3).

This paper applies the RRM model in the context of user equilibrium traffic assignment. A stochastic user equilibrium (SUE) formulation for the RRM model (its MNL model form) is presented, implemented, and tested. A mathematical problem whose solution corresponds to the SUE of an RRM-based route choice model is formulated and applied.

Several RUM-based route choice models have been developed to overcome the deficiencies of the basic RUM-MNL model form for route choice modeling, in particular to account for the similarity among overlapping routes (4). One group of models is based on the generalized extreme value theory (5), for example, the cross-nested logit model (6) and the paired combinatorial logit model (7). These models capture the similarity among routes through the structure of the error component of the utility function. Another group of models is obtained by modifying the systematic part of the utility function to account for route overlapping. This approach retains the simple closedform structure of the MNL model. Models in this group include the C-logit model (8) and the path-size logit model (9). All these models maintain the assumption of RUM-based decision making. In contrast, this paper derives the SUE formulation in the context of a new decision rule that is not utility based but regret based.

Although the RUM-SUE problem can be formulated as an optimization problem assuming that the cost function is link separable, the RRM-SUE problem cannot be formulated in the same way, because the cost function in this case is not separable. Adapting a variational inequality (VI) formulation to the RRM-SUE context solves this issue.

The rest of this paper is organized as follows. The next section presents the RRM route choice model (more specifically, its MNL-model form) and provides a brief comparison to RUM's MNL model. The subsequent section formulates the RRM-SUE problem and adapts a path-based algorithm to solve the problem. Results of its application and a comparison with RUM-SUE are illustrated for a simple grid network and for the well-known Winnipeg, Canada, network. The last section discusses the results and presents directions for further research.

## RRM ROUTE CHOICE MODEL (MNL MODEL FORM)

For ease of communication, the terms RRM and RUM are used to refer to their respective MNL model forms.

For a given network, the cost (disutility)  $c_{k,pq}$  of path k connecting origin p to destination q is generally assumed to be a linear combination of the link costs as follows:

$$c_{k,pq} = \sum_{a} \delta_{ak,pq} t_a(v_a) \tag{1}$$

where

- $\delta_{ak,pq}$  = link path indicator, which equals 1 if link *a* is a part of path *k* from *p* to *q* and 0 otherwise;
- $t_a(v_a)$  = travel time on link *a*, which is assumed to depend only on the flow on link *a*; and

 $v_a$  = flow on link a.

Although travel costs may depend on attributes other than travel time, they are used interchangeably in this paper.

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The well-known RUM route choice model expresses the route flows as follows:

$$h_{k,pq} = g_{pq} P_{k,pq} = g_{pq} \frac{\exp(-\theta c_{k,pq})}{\sum_{l} \exp(-\theta c_{l,pq})}$$
(2)

where

- $g_{pq}$  = total demand for trips between p and q in the period of analysis,
- $h_{k,pq}$  = flow on path k from p to q,
- $P_{k,pq}$  = route choice probability of path k from p to q, and l = path l.

The positive parameter  $\theta$  represents a measure of the dispersion among drivers: small values of  $\theta$  indicate a large perception variance among drivers. As  $\theta$  increases, the variability among drivers decreases, and the corresponding equilibrium flows approach those of the deterministic user equilibrium.

In an RRM formulation (1), the regret c of path k from p to q is computed by comparing the cost of this path to the costs of the other alternative routes as follows:

$$c_{k,pq} = \sum_{l \neq k} \ln \left( 1 + \exp \left[ \sum_{a} \delta_{ak,pq} t_a \left( v_a \right) - \sum_{a} \delta_{al,pq} t_a \left( v_a \right) \right] \right)$$
(3)

This formulation leaves out the random error associated with a path's regret. Various assumptions regarding the random regret term can be made. This paper focuses on the MNL form of the RRM model, in which the random regret term is distributed such that the negative of the error term has an independent and identically distributed extreme value Type I distribution. Random errors in an RRM model are formulated and interpreted differently from those in an RUM model: in an RRM model they represent unobserved regret (which in turn is a function of cost comparisons), whereas in a RUM model they directly represent unobserved costs. That is, in the RRM model the error is about perception errors not at the cost level but at the level of cost comparisons. An in-depth discussion of the rationale behind and the properties of the RRM model, along with an overview of empirical comparisons of the performance of RRM- and RUM-based models, is available elsewhere (*10*).

This formulation indicates that the regret of a specific path decreases when it compares favorably to other paths and increases when its travel costs are larger compared with alternative paths. In the case that all path costs are equal, their regrets will also be equal, and travelers will be indifferent to the choice among them. The corresponding route flows are obtained as in Equation 2 but with the regret costs given by Equation 3:

$$h_{k,pq} = g_{pq} P_{k,pq}$$

$$= g_{pq} \frac{\exp\left(-\Theta \sum_{l \neq k} \ln\left(1 + \exp\left[\sum_{a} \delta_{ak,pq} t_a(v_a) - \sum_{a} \delta_{al,pq} t_a(v_a)\right]\right)\right)}{\sum_{k} \exp\left(-\Theta \sum_{l \neq k} \ln\left(1 + \exp\left[\sum_{a} \delta_{ak,pq} t_a(v_a) - \sum_{a} \delta_{al,pq} t_a(v_a)\right]\right)\right)}$$
(4)

In the binary choice case, Equation 4 is identical to the RUM case in Equation 2. Assuming two alternatives, k and l, for each origin–destination (O-D) pair pq, the route choice probabilities are

$$P_{k,pq} = \frac{\exp(-\theta \ln(1 + \exp[c_{k,pq} - c_{l,pq}]))}{\exp(-\theta \ln(1 + \exp[c_{k,pq} - c_{l,pq}]))} + \exp(-\theta \ln(1 + \exp[c_{l,pq} - c_{k,pq}]))$$
(5)

After some manipulations, the binary logit expression is obtained:

$$P_{k,pq} = \frac{1}{1 + \left( \exp\left[ c_{l,pq} - c_{k,pq} \right] \right)^{-\theta}}$$
(6)

The detailed proof is available elsewhere (11).

The behavioral intuition behind the RRM model in Equation 3 is as follows. The traveler is assumed to compare a considered path with all other paths for their respective costs. If the considered path has a lower cost than another path with which it is compared, there is no regret. If the path with which the considered path is compared has a lower cost, then the regret for the considered path equals the difference in costs. This behavioral intuition translates into a regret function

$$c_{k,pq} = \sum_{l \neq k} \max\left\{0, \sum_{a} \delta_{ak,pq} t_{a}(v_{a}) - \sum_{a} \delta_{al,pq} t_{a}(v_{a})\right\}$$

rather than the function presented in Equation 3. However, because of the max operator, this latter function is discontinuous and therefore not differentiable around zero, which poses theoretical and practical problems for the model estimation. The logsum function used in Equation 3 provides a continuous approximation. Further discussion of this logsum form and an illustration of the close approximation it provides of the max-based formulation are available elsewhere (1).

A concise discussion is provided here of the properties of the RRM model in the single-attribute case. A choice among three parallel routes is considered, assuming that the costs of all three routes are independent of the flows. The costs on Routes A and B are 16 min and 18 min, respectively. The cost of Route C varies from 15 min to 19 min. The dispersion parameter  $\theta = 1$  in all cases. Choice probabilities are plotted for the three routes in Figure 1 for the RRM and RUM models as a function of the travel cost on Route C. The market shares computed by the RUM model are shown as solid lines, and the RRM market shares are shown as dotted lines. The results show that the RRM model predicts that when the travel costs of Route C decrease, it attracts more market share (compared with the RUM model) from the route with higher costs (B) than from the faster route (A). Furthermore, the sensitivity of the RRM choice probabilities to the travel cost is higher than that of the RUM model. When travel times on Route C are high, RUM predicts a higher share for Route C than RRM does. This trend is reversed when Route C becomes more attractive. Both these results are consistent with the general properties of the RRM model (1), which penalizes poor performance more heavily than RUM and rewards a strong performance more substantially compared with RUM models.

The higher sensitivity of the RRM model cannot be eliminated by tuning the scale of the utilities in the RUM model. To demonstrate this, Figure 2 shows the difference between the RUM and RRM models as a function of the value of the dispersion parameter  $\theta$  in the RUM model ( $\theta = 1$  in the RRM model) for the case in which the travel time on Route C is 17 min. The results suggest that for Route C, a RUM model with  $\theta = 1.15$  would generate the same choice probability as in the RRM model for Route C; an MNL model with  $\theta = 1.25$  is needed to approximate the RRM choice probability for Route A, and  $\theta = 1.5$  is needed to approximate the RRM choice



FIGURE 1 Choice probabilities generated by RUM (solid lines) and RRM (dashed lines) for three routes with different travel times.



FIGURE 2 Differences between RUM-based and RRM-based choice probabilities for each route as function of RUM dispersion parameter.



FIGURE 3 Difference between RUM choice probabilities and RRM choice probabilities for different values of theta (theta constrained to be equal for RRM and RUM).

probability for Route B. That these values differ substantially across routes suggests that the difference between RUM and RRM cannot be eliminated by tuning the dispersion parameters.

Finally, observation of the change in the differences in choice probabilities predicted by RRM and RUM as a function of the dispersion parameter is useful when the dispersion parameter is constrained to be equal across the two model types. These differences are shown in Figure 3. The figure shows that when  $\theta$  is large or close to zero, RRM choice probabilities tend to the RUM choice probabilities. This is in line with expectations: when  $\theta$  is very large, route choices become deterministic, and the fastest route is chosen by all travelers. When  $\theta$ approaches zero, route choices are fully random for both models, and the market shares are equal for all alternatives. However, for intermediate values of  $\theta$ , there are clear differences between the two model types. In this example, these differences reach a maximum of almost 10% market share approximately when  $\theta = 0.75$ .

The RRM and RUM models may yield significantly different market shares when the dispersion parameter does not have an extreme value. For a given value of  $\theta$ , RRM tends to allocate higher market shares to the best routes compared with RUM, at the expense of the worst routes.

#### **RRM-SUE PROBLEM**

#### Model Formulation

The concept of SUE was defined by Daganzo and Sheffi (11). At SUE, no driver can improve his or her perceived travel time by unilaterally changing routes. The SUE is mathematically represented as follows:

$$f_{k,pq} = g_{pq} P_{k,pq} \tag{7}$$

$$P_{k,pq} = P\left(c_{k,pq} + \varepsilon_{k,pq} \le c_{l,pq} + \varepsilon_{l,pq} \;\forall l \in K_{pq}\right) \tag{8}$$

where

$$f_{k,pq} = \text{flow on path } k \text{ connecting origin } p \text{ and destination } q;$$
  
 $\varepsilon_{k,pq} \text{ and } \varepsilon_{l,pq} = \text{random terms of paths } K \text{ and } l, \text{ respectively, connecting O-D pair } p \text{ and } q; \text{ and}$   
 $K_{pq} = \text{set of paths connecting O-D pair } pq.$ 

The first SUE models used either the simple MNL or the more complex multinomial probit as route choice models. An optimization formulation for the MNL-SUE problem was provided in the work of Fisk (12), in which the MNL route choice model gives the solution to the minimization problem. Given the nonclosed mathematical formulation for the multinomial probit, the method of successive averages was proposed to solve the multinomial probit SUE problem (13). Additional equilibrium models based on generalized extreme-value route choice models were developed by Bekhor and Prashker (14).

Because the cost function defined in Equations 1 and 2 is separable, an optimization program can be formulated. In contrast, because the RRM cost function expressed in Equation 3 is nonseparable (because of the path comparisons), an equivalent optimization program cannot be formulated. The definition of the RRM-SUE is slightly different from that of RUM-SUE—RRM-SUE refers to the situation in which no driver can decrease his or her perceived regret by unilaterally changing routes.

A VI approach is applied to formulate the RRM-SUE (15). The VI is a general problem formulation that encompasses a plethora of mathematical problems, including, among others, nonlinear equations, optimization problems, and fixed-point problems (16). In geometric

terms, the classical VI formulation states that a function  $F(x^*)$  is orthogonal to the feasible set *K* at the point  $x^*$ :

$$F(x^*)'(x-x^*) \ge 0 \qquad \forall x \in \mathbf{K}$$
(9)

This formulation is particularly convenient because it allows for a unified treatment of equilibrium problems and optimization problems. A modified formulation proposed by Zhou et al. is used in this paper (15). Let **P** represent the vector of route choice probabilities, where  $P_{k,pq}$  is defined as the RRM route choice probability as in Equation 4. The equivalent RRM-SUE model can be formulated as a VI problem, which is to find a vector  $f^* \in \Omega$  such that

$$(f - f^*)^T (f^* - P(c(f^*)) \cdot q) \ge 0 \qquad \forall f \in \Omega$$
(10)

where  $\cdot$  is the Hadamard product, that is,  $z = x \cdot y \Leftrightarrow z_i = x_i y_i$ , i = 1, 2, ..., n;  $f^*$  is a solution of the RRM-SUE model if and only if  $f^*$  is a solution of the VI problem expressed in Equation 12. The feasible set  $\Omega$  consists of the following equations:

$$q_{rs} = \sum_{h} f_{h,rs} \tag{11}$$

$$f_{h,rs} \ge 0 \tag{12}$$

The proof of the proposition follows the work of Zhou et al. (15). First, if  $f^*$  is a solution of the RRM-SUE model, from the SUE condition in Equation 7 the VI problem is satisfied naturally. Thus, any equilibrium solution of the RRM-SUE model is a solution of the VI problem. Second, suppose  $f^*$  is a solution of the VI problem; without loss of generality, fix a path h from the set of all routes connecting O-D pair (r, s) and construct a feasible flow f such that  $f_l^{mm} = f_l^{mm^*}$ ,  $(l, m, n) \neq (h, r, s)$  but  $f_h^{rs} \neq f_h^{rs^*}$ . On substituting it into Equation 10, one obtains  $(f_h^{rs} - f_h^{rs^*})^T (f_h^{rs^*} - P^{rs}_h(c^{rs}(f^*)) \cdot q^{rs}) \ge 0$ . For every effective route h between O-D pair (r, s), there should be  $f_h^{rs} > 0$ . Therefore,  $f_h^{rs^*} - P_h^{rs}(c^{rs}(f^*)) \cdot q^{rs} = 0$ . Thus, the SUE condition in Equation 7 is satisfied, and the solution of the VI problem is the solution of the RRM-SUE problem.

If the route travel cost function c(f) is continuous, then the VI formulation has at least one solution. According to the assumption of continuity, it can be seen that  $F(f)=f-P(c(f)) \circ q$  is a continuous mapping from  $\Omega$  to  $\mathbb{R}^n$ . Since  $\Omega$  is a nonempty, convex, and compact set, the VI problem has at least one solution (17).

The VI formulation for the RRM-SUE model can be written as a general form,

$$F(f)'(f - f^*) \ge 0 \qquad \forall f \in \Omega \tag{13}$$

where  $F(\cdot)$  is a general mapping from  $\Omega$  to  $\mathbb{R}^n$ . For the VI formulation in Equation 10, the mapping  $(f - P(c(f)) \circ q)$  can be represented by  $F(\cdot)$ . The preceding VI formulation belongs to a broad category of nonadditive traffic equilibrium problems (18). The RRM route choice model is nonadditive because of the path comparisons in the cost function, as in Equation 3.

Uniqueness of a solution to the VI formulation depends on the property of mapping  $F(\cdot)$ . That is, if  $F(\cdot)$  is strictly monotone, the VI formulation gives one unique equilibrium solution (16). However, the uniqueness of the RRM-SUE model may not be guaranteed because of the nonseparable route cost structure.

## Path-Based Algorithm

An algorithm to solve the RRM-SUE problem is adapted from pathbased algorithms discussed by Bekhor and Toledo (19) and applied to solve the cross-nested logit SUE problem (20). Because an optimization problem cannot be formulated, it is not possible to derive an optimal step size or to apply Armijo-type rules. Thus, the path-based algorithm with the method of successive averages is applied to find an approximate solution to the problem.

Routes are generated before the assignment and are kept fixed throughout the iterations. After performing an initial loading to obtain a feasible solution, the algorithm successively updates the travel times and path costs, calculates the RRM choice probabilities, and assigns the flows on the given routes. A predetermined step size is used to average the current path-based solution with the previous iteration (12). The path-based algorithm with the method of successive average converges to the equilibrium solution, but at the expense of a very large number of iterations (18). The equilibrium solution is achieved only if all acyclic paths are included in the path set. Because this is not practical for real networks, a suboptimal solution is achieved. If the path set is fixed, as in this paper, this solution is unique.

Because of the nonoptimized step size, the algorithm needs a large number of iterations to reach convergence. The stopping criterion for the algorithm is based on the internal inconsistency of the solution:

$$RMSE^{(n)} = \sqrt{\frac{1}{k} \sum_{rs} \sum_{k} \left( h_{k,rs}^{(n)} - f_{k,rs}^{(n)} \right)^2}$$
(14)

where

RMSE = root mean square error,

K = number of routes in the choice sets,

n = iteration counter,

 $h_{k,rs}$  = path flow computed according to the route choice model for given travel times, and

 $f_{k,rs}$  = current path flow on the network.

## RESULTS

#### Grid Network

Figure 4 shows a simple grid network. The free-flow travel times and link capacities are respectively indicated in the network. In this example, there are two O-D pairs with positive demand: between 1 and 6 (10 units of flow) and between 1 and 9 (20 units of flow).



FIGURE 4 Grid network.

For this simple network, the universal choice set can be generated. It is composed of three routes for O-D pair 1-6 and six routes for O-D pair 1-9. A path-based algorithm with the method of successive averages was used for all models, and the stopping criterion was set to 0.001 maximum RMSE difference between link flows. The following link performance function was used in all the tests:

$$t_{a} = t_{0a} * \left( 1 + 0.6 \left( \frac{x_{a}}{s_{a}} \right)^{4} \right)$$
(15)

where

 $t_a$  = travel time on link a,

 $t_{0a}$  = free-flow travel time on link a,

 $x_a$  = flow on link a, and

 $s_a$  = capacity on link a.

It is assumed that path travel times are obtained by summing the travel times of each link that forms the path. The flow  $x_a$  is obtained after assignment of the path flows for each O-D pair with positive flow on the network.

This simple example illustrates the differences between the RRM and RUM equilibrium results. The uncongested fastest route between 1 and 9 is Route 1-4-5-6-9, and the uncongested fastest route between 1 and 6 is Route 1-4-5-6. Figure 5 shows a comparison of the path flow probabilities of choosing Route 1-4-5-6-9 according to RUM-SUE and RRM-SUE network equilibrium results as a function of the dispersion parameter  $\theta$ . The solid lines represent RUM-SUE results for different demand levels, and the dashed lines represent RRM-SUE results for different demand levels. The total demand for each O-D pair is scaled by constant factors (0.6, 0.8, 1.0, 1.2, and 1.4), and for each demand level the equilibrium is computed.

For a given demand, the probability of choosing Route 1-4-5-6-9 increases with  $\theta$ . This result is consistent with the theory, because high values of  $\theta$  indicate low variance for travel time perception. For a given  $\theta$ , the probability of choosing Route 1-4-5-6-9 decreases for increasing demand. This result is also expected, because as the network becomes more congested, the path travel times tend to be close to each other, lowering the relative attractiveness of Route 1-4-5-6-9. However, the proportion of flow in this route is always higher than 1/6. This extreme case occurs only when  $\theta$  is zero, meaning that the travel time variance tends to infinity, and therefore the probability of choosing any one of the six routes between Origin 1 and Destination 9 is equal. In line with expectations and the numerical examples, RRM-SUE results in higher shares for Route 1-4-5-6-9 than RUM-SUE for all demand levels. Route 1-4-5-6-9 is the fastest route for this O-D pair, and RRM rewards this path more strongly than RUM. However, the difference between RRM flow and RUM flow decreases for increasing demand. This result is specific for the presented grid network, because the travel times on the alternative routes become close to the fastest route for increasing demand levels. Similar results are obtained for Route 1-4-5-6.

#### Winnipeg Network

The database of the network of Winnipeg, Manitoba, Canada, provided in the EMME/2 software (21) is used to compare RUM-SUE and RRM-SUE results for a more realistic network. The network is composed of 948 nodes (154 of which are centroids), 2,535 links, and 4,345 O-D pairs with positive demand. The total demand on the network is 54,459 trips for the morning peak. The volume-delay function for each link is based on the Bureau of Public Roads formula with link-specific parameters, calculated from the original EMME/2 data.



FIGURE 5 Choice probabilities for Route 1-4-5-6-9 for RUM-SUE (solid lines) and RRM-SUE (dashed lines) and for different demand scale levels.

Type of Flow and Maximum Number of Routes <sup>a</sup>	K	Theta					
		0.01	0.05	0.1	0.5	1	
Path							
50	174,491	0.297	0.528	0.642	0.936	0.976	
5	21,723	0.280	0.567	0.740	1.054	1.156	
Link							
50	2,535	136.765	143.777	126.271	71.756	52.742	
5	2,535	24.481	30.918	30.435	22.787	18.289	

TABLE 1 RMSE between RUM-SUE and RRM-SUE Path Flows and Link Flows

<sup>a</sup>Generated for each O-D pair.

Routes were generated before the assignment with a combination of the link elimination method (22) and the penalty method (23) and with a penalty of 5% increased travel time on the shortest path links. Only acyclic paths were considered in these methods. A total of 174,491 unique routes were generated for all O-D pairs (average of 40.1 routes per O-D pair). The maximum possible number of routes generated for each O-D pair was 50. Inspection of the routes generated for the O-D pairs reveals that the choice set used for the analysis includes both completely disjointed routes and very similar routes. This was expected because of the methods (link penalty and link elimination) chosen to generate the routes: the link elimination method produces disjoint routes (because of the removal of all links belonging to the shortest path), and the link penalty method produces similar routes because of the low penalty (5% increased link travel time) used to find the subsequent routes. The same choice set was used in previous papers (24, 25).

Table 1 shows the effect of the values of the parameter  $\theta$  on the RMSE of the difference between RUM-SUE and RRM-SUE. The deviation between the two models in Table 1 is measured as follows:

$$RMSE = \sqrt{\frac{1}{K} \sum_{rs} \sum_{k} \left( f_{k,rs}^{(RUM)} - f_{k,rs}^{(RRM)} \right)^2}$$
(16)

The same formula is used to calculate the deviation at the level of link flow. The table includes results computed for a more restrictive case allowing a maximum of five routes per O-D pair. In this case, 21,723 routes are generated of 21,725 possible (4,435 \* 5).

The results presented in Table 1 indicate that the path flow deviation increases with  $\theta$ . The average demand on a route is about 0.31 for up to 50 routes per O-D pair (54,459/174,491), and about 2.51 for up to five routes per O-D pair. This means that the RMSE for the five-route case is relatively small compared with the 50-route case. Nevertheless, in both cases the deviation is high, meaning that RUM-SUE and RRM-SUE produce significantly different path flows.

In contrast to the path flows, the RMSE link flows do not exhibit a monotonic pattern. This result is difficult to interpret, because many routes have several links in common. RMSE values are higher for the 50-route case than for the five-route case. Following the path flow results, in both choice sets the differences between the two models are significant.

Figure 6 shows the link flow difference between RRM-SUE and RUM-SUE results, setting  $\theta$  equal to 0.5. Green indicates that RUM link flows are higher than RRM link flows. There is a concentration of RUM link flows around the city center. This is explained by the relatively high number of low-capacity links in the city center; consequently, more congested routes pass through the center. Similar

to the three-route example and the grid network example, the RRM route choice model more heavily penalizes the more congested routes, in comparison with the RUM route choice. Therefore, the overall link flow pattern results in more RUM flows in the city center. This interpretation is similar to that of other comparisons of RUM-based cross-nested logit SUE and MNL-SUE link flows (25).

The computation times for a single iteration of RRM-SUE and RUM-SUE are quite similar because the additional effort related to path comparisons in RRM-SUE is not time-consuming. On a desktop PC computer (Intel Core 2 Duo CPU, 3.0 GHz speed, and 4.0 GB RAM), the computation took 2.2 s per iteration. However, RRM-SUE requires more iterations than RUM-SUE to converge. Table 2 gives the number of iterations needed to reach convergence for two criteria (RMSE equal to 0.1 and 0.01, respectively) and for different values of theta with the Winnipeg network and five routes per O-D pair.

### SUMMARY AND FURTHER RESEARCH DIRECTIONS

This paper discussed the RRM approach to route choice modeling and presented a VI formulation for the RRM-SUE model. The results show that the model can be implemented on real-size networks in practice.

The comparison between RUM-SUE and RRM-SUE results, performed for a simple network and for a real-size network, indicates that differences among the equilibrium route flows can be significant and are in line with the differences in behavioral premises underlying the two model paradigms. Depending on the network topology and the number of routes generated, the results may be quite different, even at the link flow level.

This study compared the results between RRM and RUM models of the MNL form. Further research is needed to compare RRM and RUM in the context of other route choice model forms, such as C-logit, path-size logit, or cross-nested logit models, and to examine their effects on the equilibrium solutions. In addition, it would be interesting to explore how similarity and route overlap can be modeled in an RRM framework. Other issues may affect the performance of the solution algorithm and equilibrium flow patterns, such as various demand levels and different generation methods for route sets (a priori or column generation). The effects of these issues are worth further investigation. The convergence properties, such as robustness and efficiency, of path-based algorithms for solving equilibrium problems can be compared in future research.

In addition to these theoretical advances, a direction for further research would be to provide additional empirical testing between RRM-based and RUM-based route choice models. Data should be



FIGURE 6 Link flow difference between RRM-SUE and RUM-SUE models ( $\theta = 0.5$ ).

collected and analyzed at the level of the individual traveler's choices (by using stated preference surveys or revealed preference data sets), as well as at the level of aggregate network flows.

The route choice model considered in this article is a function of travel times only. The formulation of the problem can accommodate additional explanatory variables, similar to the generalized cost variable in deterministic traffic assignment problems. More complex utility functions are yet to be implemented in traffic assignment models. However, the RRM model can be easily formulated at the multiattribute level—its original formulation is multiattribute. The assumption in a multiattribute setting is that attribute-level regrets are summed over all attributes, so that associated parameters reflect the relative importance of corresponding attributes. In this regard, the multiattribute RRM model resembles the multiattribute RUM model (more specifically, its linear-additive model form), which also assumes that attribute-level utilities.

#### TABLE 2 Iterations Needed to Converge

Convergence Criterion (RMSE)	Model	Theta					
		0.01	0.05	0.1	0.5	1	
0.1	RRM	5	19	31	132	242	
0.1	RUM	3	10	17	62	127	
0.01	RRM	39	178	301	1,261	2,348	
0.01	RUM	14	90	155	595	1,209	

The results presented here are based on several assumptions common to simple equilibrium models: static assignment, fixed demand, separable volume-delay function, and single-user class. Additional research is needed to extend and verify the RRM-SUE model for more general problems.

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