A Modified Loosely-Coupled Approach

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Abstract

A popular INS/GPS integration strategy is the loosely-coupled approach. Its main disadvantage lies in the inability to provide measurement updates during periods of partial GPS availability. In such cases, the INS navigation solution drifts over time. This paper proposes construction of fictitious GPS satellites in order to facilitate GPS receiver position and velocity solutions in cases of partial GPS availability. The paper demonstrates the contribution of the proposed approach via several field experiments. There, introduction of the synthetic GPS measurements greatly reduced the navigation errors obtained by the standalone INS, and facilitated the classical implementation of the loosely coupled integration.

Keywords: Loosely-Coupled Approach, Fictitious-GPS, Vehicle Constraints

1. Introduction

GPS and INS integration aims to utilize the advantages of the two individual systems and overcome their weaknesses. To achieve this, four types of GPS/INS coupling architectures have been proposed: *i*) uncoupled, *ii*) loosely coupled, *iii*) tightly coupled, and *iv*) ultra-tightly (deep) coupled. In uncoupled integration (Titterton and Weston 2004), the GPS position and velocity estimates are used to reset their INS counterparts at regular time intervals. In loosely coupled (LC) integration both the GPS and INS function autonomously and then a GPS/INS integrated solution is derived (Grewal et al. 2007). The LC approach is a decentralized GPS/INS filter since the raw GPS observables are inserted to the standalone GPS filter to obtain position and velocity. The GPS position and velocity estimates and the corresponding INS position and velocity are compared, and the resulting differences form the measurement inputs to the GPS/INS filter. In contrast, the tightly coupled (TC) approach forms a GPS/INS centralized filter which does not separate GPS and INS navigation solutions. Instead, a single integration filter is employed to fuse INS and GPS measurements (Bullock et al. 2006) and form a blended GPS/INS solution. The raw GPS output pseudorange and pseudorange rate measurements and those constructed from the INS prediction are combined to form the measurements used in the estimation filter (Greenspan 1996). This scheme provides a more accurate solution than LC as the raw GPS observables (pseudorange and pseudorange rate) are introduced to the single INS/GPS filter (Alban et al. 2003; Hide and Moore 2005; Petovello et al. 2004). Finally, the ultra-tight integration combines GPS signal tracking and INS/GPS integration into a single estimation filter. In contrast with other integration schemes, where the GPS aids the INS, in this approach the INS conceptually aids the GPS. Although this approach offers faster GPS signal reacquisition and multipath resistance (Titterton and Weston 2004), it can only be realized using special hardware components and requires access to the firmware of the receiver.

The most common integration scheme in practice is the LC approach (Godha and Cannon 2007; Groves 2008). It offers greater flexibility and modularity in terms of system implementation as it allows use of off-the-shelf hardware that can be easily assembled without major development. LC integration also offers high system robustness, due to the independent solutions it produces: standalone GPS and INS, followed by an integrated solution. In addition, it is suitable for any INS and GPS receiver, and can be used to retro-fit applications (Titterton and Weston 2004). However, it requires four or more satellites to form a GPS position/velocity solution, which in turn is introduced to the decentralized GPS/INS filter. If less than four satellites are available, the navigation solution will rely solely on the standalone INS solution which, regardless of it grade, drifts in time (Farrell and Barth 1999).

This paper proposes a modification to the LC approach that enables it to be used when less than four satellites are available. The modification is based on introduction of fictitious GPS (FGPS) satellites to bring the total of available satellites to at least four. The FGPS are virtual satellites that are positioned at space in a desired location. The synthetic pseudorange measurements associated with FGPS satellites are derived mathematically using the INS estimates of the platform position and velocity.

The proposed Modified Loosely Coupled (MLC) methodology enables application of the LC integration while requiring only small adjustments in the navigation software and no hardware modifications. It offers an alternative to TC when less than four GPS satellites are in view. With the proposed methodology at hand, the designer may choose between it and TC implementation. Although, theoretically TC performance is better, the MLC may be preferred if separate solutions or simple hardware handling are required.

The rest of the paper is organized as follows: Section 2 introduces the basic principles of the LC integration. Section 3 describes the concept of FGPS satellites and discusses their creation and positioning. Section 4 demonstrates the proposed approach using data that was collected in several field experiments. The contribution of vehicle constraints, which are derived from a priori knowledge about the system/vehicle dynamics, is also assessed. Section 5 discusses the conclusions of this research.

2. GPS/INS Loosely Coupled Approach

The INS mechanization equations provide no information about errors in the system states as they process raw data from the Inertial Measurement Unit (IMU) to estimate navigation parameters. The IMU outputs contain additional errors that cannot be compensated for. To improve the INS performance, it is necessary to develop an error model which describes how the IMU sensor errors propagate into navigation errors through the motion equation. These navigation errors are then corrected for in order to obtain an improved navigation solution. Several models (e.g. Titterton and Weston 2004; Jekeli 2000) were developed to describe the time-dependent behavior of these errors. The classic approach is perturbation analysis, in which navigation parameters are perturbed with respect to the true navigation frame. Perturbation is implemented via a first-order Taylor series expansion of the states. A complete derivation of this model can be found in Britting (1971) and Shin (2001). The error state vector of such model is defined as

$\delta x = \begin{bmatrix} \delta r^n & \delta v^n & \varepsilon^n & \delta b_a & \delta b_g \end{bmatrix}^T, \delta x \in R^{15}$ and consists of position error, velocity error, attitude errors, and accelerometer and gyro bias/drift. A detailed description of the parameters of the corresponding state-space model can be found in (Farrell 2008).

We incorporate the INS dynamics with GPS (real and fictitious) aiding in a Kalman filter (Zarchan and Musoff 2005). In LC integration, the position and velocity estimates from the GPS are inserted as measurements to the Kalman filter, which estimates the INS errors states. The measurement equation is given by:

$$z_{LC} = \begin{bmatrix} r_{INS} - r_{GPS} \\ v_{INS} - v_{GPS} \end{bmatrix} \tag{1}$$

where r_{INS} and v_{INS} are the INS position and velocity solution, respectively; and r_{GPS} and v_{GPS} are the GPS position and velocity solution, respectively. The corresponding measurement matrix is given by:

$$H_{LC} = \begin{bmatrix} I_3 & 0_3 & 0_3 & 0_3 & 0_3 \\ 0_3 & I_3 & 0_3 & 0_3 & 0_3 \end{bmatrix} \tag{2}$$

3. Modified Loosely Coupled Methodology

As noted above, when less than four satellites are available the LC approach cannot be used. To overcome this problem we propose the MLC appraoch. A simplified flow-chart of the MLC approach is presented in Figure 1(a). Using the INS derived position and velocity and data on unavailable GPS satellites, fictitious GPS (FGPS) satellites are created. Using the FGPS and the available satellites, GPS position and velocity can be estimated. The INS and GPS position and velocity are then inserted to the Kalman filter, in a similar fashion as with the LC scheme. For a comparison of the proposed approach to the LC and TC integration, simplified flowcharts of both approaches are presented in Figure 1(b)-1(c) respectively. The next section describes how FGPS satellites are generated and utilized, together with the available satellites, to calculate GPS position and velocity.

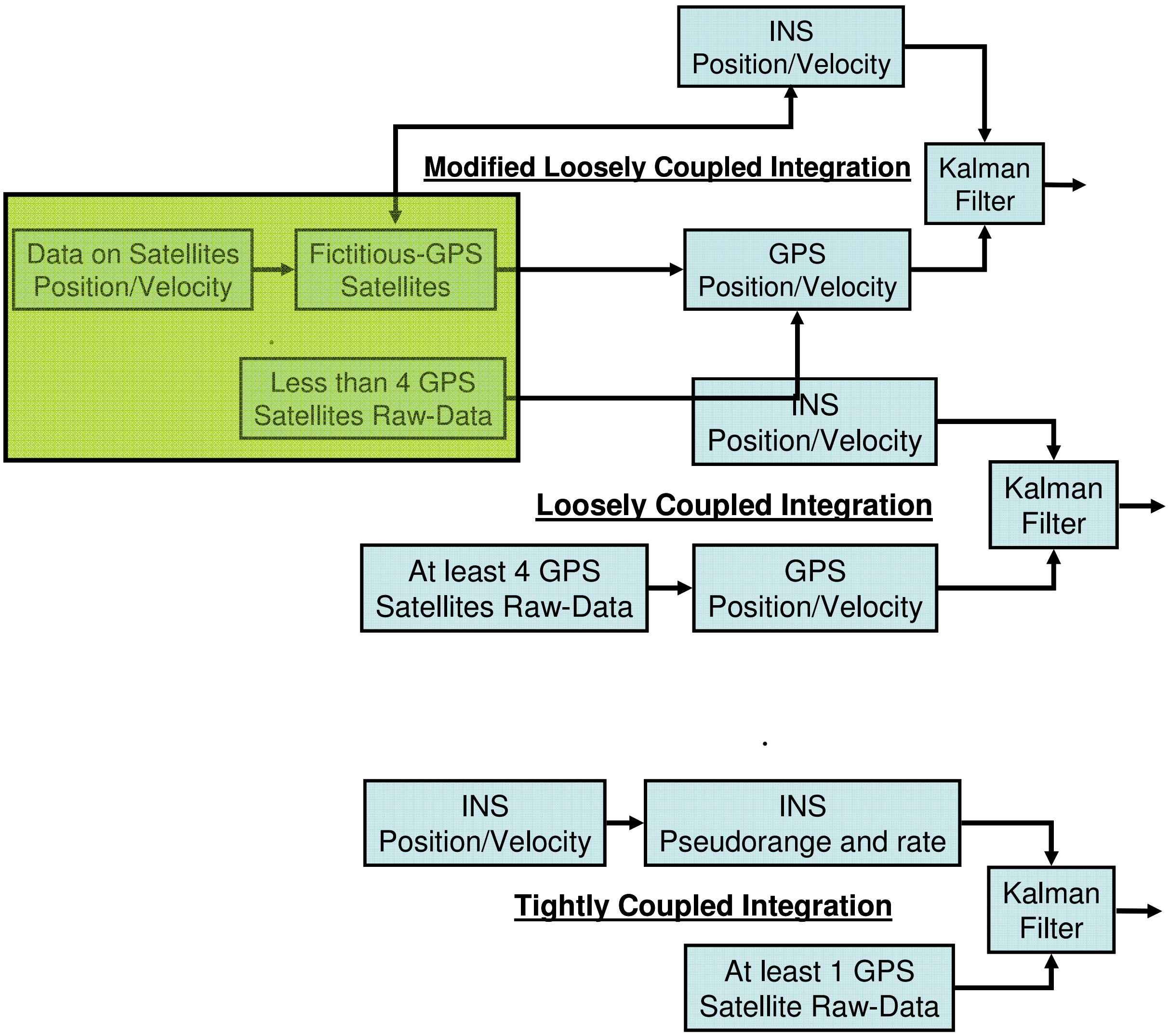

Figure 1: Schematic block diagrams of INS/GPS integration approaches, a) the proposed MLC, b) LC, and c) TC.

3.1 Fictitious Satellites

Suppose $1 \le n < 4$ satellites are acquired by the receiver, then at least $m = 4 - n$ FGPS satellites need to be created. The position, $p_j = \begin{bmatrix} x_j & y_j & z_j \end{bmatrix}^T$, and velocity, $v_j = \begin{bmatrix} vx_j & vy_j & vz_j \end{bmatrix}^T$,of each $j = 1, \ldots, m$ FGPS is assumed known. We discuss the positioning of the satellites in the next section. Once the positions and velocities have been set, they are combined with the INS derived position, $p_{INS} = \left[x_{INS} \quad y_{INS} \quad z_{INS}\right]^T$, and velocity, $v_{INS} = \left[vx_{INS} \quad vy_{INS} \quad vz_{INS}\right]^T$ to obtain the FGPS synthetic pseudorange, $\tilde{\rho}_j^{FGPS}$, and rate, $\dot{\tilde{\rho}}_j^{FGPS}$, of each *j*-th satellite:

$$\tilde{\rho}_j^{FGPS} = \left\| \overline{p}_j - \overline{p}_{INS} \right\| \tag{3}$$

$$\dot{\tilde{\rho}}_j^{FGPS} = \frac{\left(\overline{p}_j - \overline{p}_{INS}\right)\cdot\left(\overline{v}_j - \overline{v}_{INS}\right)}{\tilde{\rho}_j^{FGPS}} \tag{4}$$

Note that the position and velocity of GPS satellites used in Eqs (3)-(4), are from FGPS satellites which are not available to the receiver. In the TC approach, the same equations are applied, only to the available satellites in view.

The initial FGPS satellites position and velocity are propagated via Kepler motion equations (Battin 1999):

$$\ddot{r} + \frac{\mu r}{\left\|r\right\|^3} = p\left(r,t\right) \tag{5}$$

where μ is the gravitational parameter and $p\left(r,t\right)$ denotes the perturbations acting upon the satellite. For simplicity all perturbations on the satellites orbit are neglected, including the perturbing acceleration due to the zonal harmonics, J_2, by setting $p\left(r,t\right) = 0$. Note that for real GPS satellites the accuracy of the satellites position at the time the measurement was received is critical for the estimation accuracy, and therefore broadcast ephemeris is used. However, for FGPS satellites, the accuracy in their location is meaningless since there is no real measurement and therefore no measurement noise. The systematic measurement matches exactly the assumed FGPS satellite positions (see Eq. (19)). Thus, high orbit accuracy is not required.

3.2 Initial Selection of the FGPS Satellites

To fully define the approach, a method to select appropriate positions and velocities for FGPS satellites needs to be specified. To that end, we examine two strategies for choosing the FGPS satellites. Both are based on the Dilution of Precision (DOP) of FGPS and real satellites:

(1) DOP FGPS Satellites: A first approach is to create FGPS satellites based on the real satellites that are not acquired by the receiver. With this approach, the initial position and velocity for each FGPS satellite can be calculated using almanac data. The selection of FGPS satellites from the set of GPS satellites that are not available to the receiver is based on minimizing the DOP of the selected FGPS and in-view GPS satellites. An algorithm to find this selection is presented in (Conley el at. 2006).

(2) Artificial GPS Satellites: It was shown (Parkinson and Spiker 1996) that the configuration that yields minimum DOP is tetrahedron with an equilateral triangle as its base. That is, one satellite at the zenith of the user and other satellites equally spaced in a plane perpendicular to the user link to the satellite zenith. Therefore, as a substitute or in addition in addition to finding FGPS satellites based on the DOP, it is possible to calculate based on the INS position the four locations of the tetrahedron and create in each of those locations an artificial GPS satellite (which does not exist in reality). If used in addition to the DOP method, a total of 8 satellites are used. This strategy is denoted as ARTDOP.

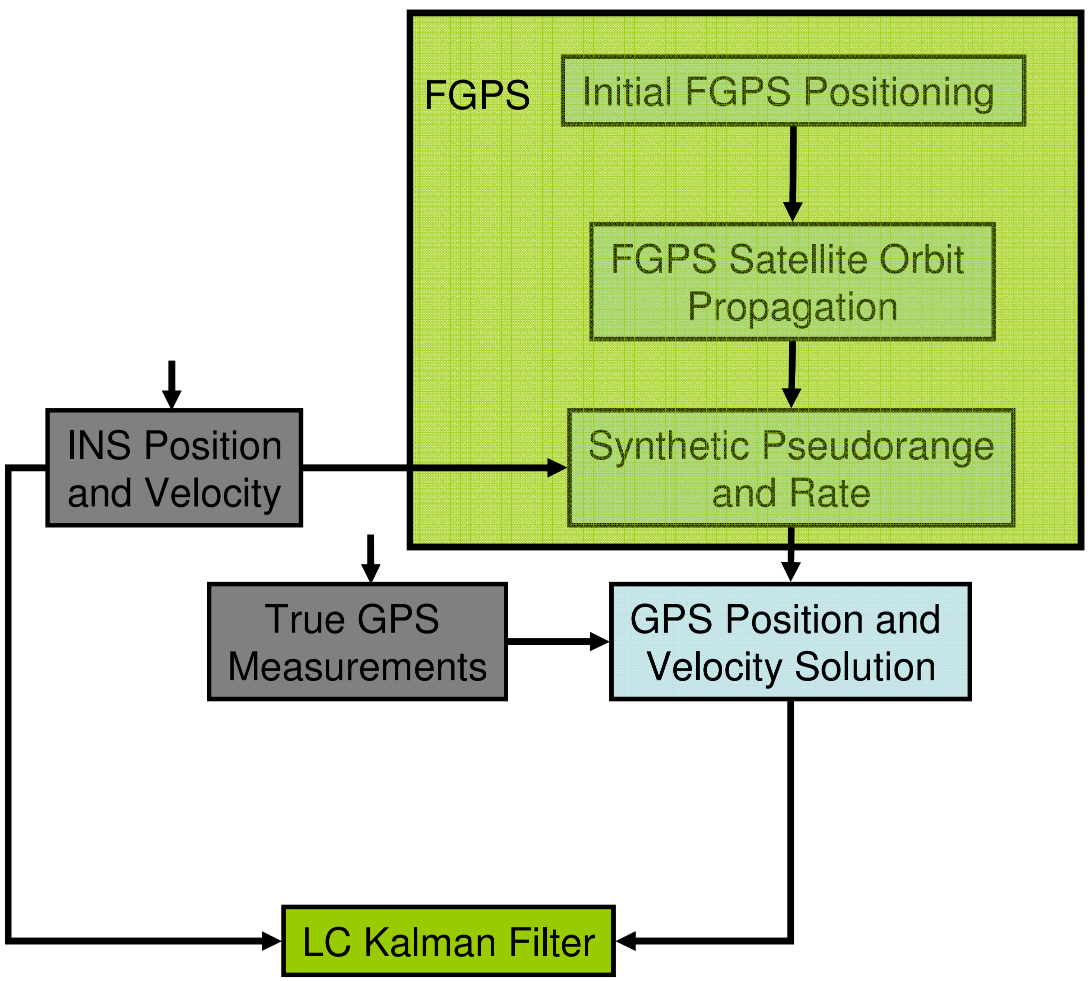

Figure 2: Creating FGPS satellites

3.3 Vehicle Constraints

Further enhancement of the DOP strategy is developed by incorporating vehicle constraints to the estimation process. These constraints translate a priori knowledge of the system into measurements to the estimator. For land navigation, Dissanayake et al. (2001) utilized the fact that, normally, vehicles do not slip or jump off the ground as a pseudo-measurement of vehicle velocity, and recently, Shin (2001) ,Godha and Cannon (2007) and Syed et al. (2008) demonstrated the use a velocity pseudo-measurement as aiding to a linear INS error model by perturbing the velocity governing equation.

Two types vehicle constraints are addressed: the body-velocity and constant-height (denoted *VC* and *HC* respectively). A body velocity constraint utilizes the realization that vehicles travel on the ground and do not slide on it. It is assumed that the body axes coincide with the axes of the IMU axis. The definition of the body frame is as follows: x_B axis pointing towards the front of the vehicle, y_B axis pointing towards the right of the vehicle and z_B Completes the right hand triad. Thus, velocities in the body frame along the y_B and z_B directions can be assumed almost zero (Shin 2001), namely $v_{B_y} \cong 0$ and $v_{B_z} \cong 0$. Under these assumptions, the computed velocity in the body frame can be expressed as

$$v^b = \left(T^{b\to n}\right)^T v^n \tag{6}$$

Perturbing Eq. (6) and rearranging it, leads to

$$\delta v^b = T^{n\to b}\delta v^n - T^{n\to b}\left(v^n \times\right)\delta\varepsilon^n \tag{7}$$

where $\left(v^n \times\right)$ is the skew symmetric form of the velocity vector. From the second and third rows of the previous vector equation, the measurement equations can be constructed as:

$$z_k = \begin{bmatrix} v_{B_y} \\ v_{B_z} \end{bmatrix}_{INS} - \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} v_y \\ v_z \end{bmatrix} \tag{8}$$

$$H_k = \begin{bmatrix} 0_{1\times3} & T_{12}^{b\to n} & T_{22}^{b\to n} & T_{32}^{b\to n} & v_E T_{32}^{b\to n} - v_D T_{22}^{b\to n} & v_D T_{12}^{b\to n} - v_N T_{32}^{b\to n} & v_N T_{22}^{b\to n} - v_E T_{12}^{b\to n} & 0_{1\times6} \\ 0_{1\times3} & T_{13}^{b\to n} & T_{23}^{b\to n} & T_{33}^{b\to n} & v_E T_{33}^{b\to n} - v_D T_{23}^{b\to n} & v_D T_{13}^{b\to n} - v_N T_{33}^{b\to n} & v_N T_{23}^{b\to n} - v_E T_{13}^{b\to n} & 0_{1\times6} \end{bmatrix} \quad (9)$$

where v_y and v_z are the measurement noise value for compensating possible discrepancies in the assumptions made on the zero body velocity.

The second constraint is applied to the height component (Klein et al. 2010). Usually, when driving in an urban environment, the height remains almost constant for short time periods. Assuming constant height $h = h_c$ and thus $v_D = 0$, the measurement equations can be constructed as:

$$z = \begin{bmatrix} h_{INS} - h_c \\ v_D - 0 \end{bmatrix} + \begin{bmatrix} v_h \\ v_{vd} \end{bmatrix}, \quad H = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0_{1\times8} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0_{1\times8} \end{bmatrix} \quad (10)$$

where v_h and v_{vd} are the measurement noise value for compensating possible discrepancies in the assumptions made on the constant height and zero velocity in the down direction.

4 Analytical Assessment

In this section we establish a theoretical foundation for the FGPS methodology. To that end, we consider a two dimensional case (altitude y-axis and downrange x-axis) were only two satellites are required to estimate receiver position (with no clock bias). Suppose, only one satellite (known satellite position $\begin{bmatrix} x_1^s & y_1^s \end{bmatrix}$) with pseudorange, $\tilde{\rho}_1$, is viewed by the receiver . Therefore the receiver position, $\begin{bmatrix} x_k & y_k \end{bmatrix}$, cannot be determined. When an INS is available, its estimated position is also available. We denote this position $\begin{bmatrix} \tilde{x}_{INS} & \tilde{y}_{INS} \end{bmatrix}$. Using the approaches described in Section 3.2 we generate an FGPS satellite at position $\begin{bmatrix} x_2^s & y_2^s \end{bmatrix}$. Employing the INS solution and FGPS satellite position we obtain the synthetic (calculated) measurement $\tilde{\rho}_2$

$$\tilde{\rho}_2 = \sqrt{(\tilde{x}_{INS} - x_2^s)^2 + (\tilde{y}_{INS} - y_2^s)^2} \quad (11)$$

where tilde stands for calculated quantity.

The receiver position can now be estimated as the solution to the system of two measurement equations:

$$\tilde{\rho}_i = \sqrt{(x_k - x_i^s)^2 + (y_k - y_i^s)^2}; i = 1, 2. \tag{12}$$

For simplicity and without loss of generality, through proper shift and rotation of the coordinate system, we assume that $x_k = y_k = 0$ and $y_1^s = y_2^s = y^s$. Using the second assumption, the solution to the platform downrange coordinate, $\hat{x}_k$ is given by:

$$\hat{x}_k = \frac{\left[(\tilde{\rho}_1)^2 - (\tilde{\rho}_2)^2\right] + \left[(x_2^s)^2 - (x_1^s)^2\right]}{2(x_2^s - x_1^s)} \tag{13}$$

To evaluate the algebraic expression in Eq. (13) we rewrite the measurements as $\tilde{\rho}_1 = \rho_1 + \delta\rho_{GPS}$ where ρ_1 is the true pseudorange and $\delta\rho_{GPS}$ is the measurement error. Similarly, we denote the synthetic measurement by $\tilde{\rho}_2 = \rho_2 + \delta\rho_{INS}$ where $\delta\rho_{INS}$ is the error resulting from the INS position estimate error. Substituting these definitions into Eq. (13) we get that the error in the x-coordinate of the position estimate is given by:

$$\delta\hat{x}_k = \frac{(\tilde{\rho}_1\delta\rho_{GPS} - \tilde{\rho}_2\delta\rho_{INS})}{(x_2^s - x_1^s)} \tag{14}$$

As expected the accuracy of the downrange solution depends on the INS error and the FGPS satellite location. For an ideal GPS and INS measurements ($\delta\rho_{GPS} = \delta\rho_{INS} = 0$) Eq. (14) reduces zero. The position of the fictitious satellite x_2^s has an important affect on the accuracy of the error in the estimated receiver position.

Next, we estimate the term $\delta\rho_{INS}$ in order to compare the MLC solution in Eq. (14) with the standalone INS solution. We define the basic INS estimates as:

$$\begin{aligned} \hat{x}_k^{INS} &= x_k + \delta x_{INS} \\ \hat{y}_k^{INS} &= y_k + \delta y_{INS} \end{aligned} \tag{15}$$

Where, $\hat{x}_k^{INS}$ and $\hat{y}_k^{INS}$ are the receiver coordinates estimated by the INS. δx_{INS} and δy_{INS} are the corresponding errors.

Plugging $\hat{x}_k^{INS}$ and $\hat{y}_k^{INS}$ from Eq. (15) into Eq. (11) and using a linear Taylor expansion we get:

$$\delta\rho_{INS} \cong \frac{\delta x_{INS}(x_k - x_2^s)}{\tilde{\rho}_2} + \frac{\delta y_{INS}(y_k - y^s)}{\tilde{\rho}_2} \tag{16}$$

Using the assumption that the receiver is at located at (0, 0) and further assuming that the magnitude of the INS error along the two axes is similar ($\delta y_{INS} \approx \delta x_{INS}$) and substituting Eq. (16) into Eq. (14) the estimation error for the x-axis is given by:

$$\delta x_k = \frac{(\tilde{\rho}_1 \delta\rho_{GPS})}{(x_2^s - x_1^s)} + \frac{\delta x_{INS}(x_2^s + y^s)}{(x_2^s - x_1^s)} \tag{17}$$

For an ideal GPS ($\delta\rho_{GPS} = 0$) the error reduces to:

$$\delta x_k = \frac{\delta x_{INS}(x_2^s + y^s)}{(x_2^s - x_1^s)} \triangleq \delta x_{INS}\alpha \tag{18}$$

The position error along the x-axis depends on the choice of location of the FGPS satellite. Thus, x_2^s can be chosen such that $\alpha < 1$, namely causing the estimation error to be lower than that of the standalone INS. In particular, if $x_2^s = -y^s$, the error is nullified.

In addition, the positioning of the FGPS satellite is made by minimizing the DOP – thus we shall calculate x_2^s following this approach. To that end, we evaluate the Jacobian of the system of estimated pseudoranges for the various satellites that were defined in Eq. (11).

$$H = \begin{bmatrix} \frac{\partial\tilde{\rho}_1}{\partial x_k} & \frac{\partial\tilde{\rho}_1}{\partial y_k} \\ \frac{\partial\tilde{\rho}_2}{\partial x_k} & \frac{\partial\tilde{\rho}_2}{\partial y_k} \end{bmatrix} \tag{19}$$

and define $V = (H^T H)^{-1}$. We minimize the DOP by

$$\arg\min_{x_2^s} \left\| trace(V) \right\| \tag{20}$$

yielding

$$x_2^s = -\left(y^s\right)^2 / x_1^s \tag{21}$$

Substitution Eq. (21) into Eq. (18) gives

$$\delta x_k = \delta x_{INS} \left[\frac{y^s(y^s - 1)}{\tilde{\rho}_1^2} \right] \tag{22}$$

Since $\tilde{\rho}_1^2 = (y^s)^2 + (x_1^s)^2$, the expression in the brackets is smaller than one thus reducing the standalone INS error regardless of the location of the actual satellite in view.

Next, we examine Kalman filter performance with the MLC approach when only the downrange coordinate is introduced as aiding. In this case the measurement equation (Eq. (1)) reduces to:

$$z = x_{INS} - x_{GPS} = x_k + \delta x_{INS} - (x_k + \delta x_{GPS}) = \delta x_{INS} - \delta x_{GPS} \tag{23}$$

Suppose that two ideal GPS satellites where in view thus $\delta x_{GPS} = 0$ and Eq. (23) will read

$$z = \delta x_{INS} \tag{24}$$

which is the best performance the LC approach can provide since the measurement is exactly the INS error. On the other hand, if only one satellite was available the LC approach could not be used since in that case δx_{GPS} is not defined. In the MLC approach δx_{GPS} equals to the right-hand-side of Eq. (22) when the true downrange coordinate is zero. Introducing Eq. (24) into Eq. (23) we have

$$z = \delta x_{INS} (1 - \alpha) \tag{25}$$

This time we insert to the measurement the INS error multiplied by some factor. As expected we observe that MLC approach performs worse then the ideal LC but performs much better from the case when LC cannot be applied – i.e. the standalone INS

In order to apply the MLC methodology at least one true satellite must be in view. This can be easily explained when addressing the one dimensional case; there a single satellite is enough for obtaining receiver position. For example, if indeed one true satellite was in view the receiver position would be $x_k = \tilde{\rho}_1 + x_1^S$. If no satellite was available applying MLC approach we find the synthetic measurement based on the INS solution $\tilde{\rho}_1 = x_{INS} - x_1^S$. Then the receiver solution is $x_k = \left(x_{INS} - x_1^S\right) + x_1^S = x_{INS}$. Namely the receiver solution is identical to the INS solution, adding no knew information.

5 Analysis & Discussion

We demonstrate the contribution of the proposed FGPS strategies with a case study of driving in an urban environment. The trajectory data was with MEMS INS/GPS. The vehicle was equipped with a Microbotics MIDG II (Microbotics, 2010) INS/GPS system. Noise densities of the acceleration and angular rate are $150\mu g/\sqrt{Hz}$ and $0.05\left(\deg/\sec\right)/\sqrt{Hz}$ respectively. Three trajectories, featuring diverse road/driving characteristics, are evaluated in the analysis. In the first trajectory, the stationary vehicle accelerated to $v = 60\left[km/h\right]$, and then kept a velocity in the range between 60 and 80 [*km/h*]. Relief in this experiment had height variations of ~15 [m] along the trajectory. The examined trajectory included left and right turns. In the second experiment the road was unpaved, with many pits along it. Height variations along the road were ~25[m]. Finally, in the third experiment the vehicle traveled uphill a path with height change of ~120 [m] along the trajectory. All three trajectories feature challenging scenarios which exceed the more typical GPS outage scenario, where the line-of-sight to satellites is blocked when driving along a straight road with dense high-rise buildings surrounding it.

To evaluate the contribution of the MLC approach, position and the velocity error measures are examined. To that end, the position and velocity vectors are transformed into e-frame coordinates, and, the following error measure is utilized:

$$\varepsilon_q(t) = q_{aiding}(t) - q_{nominal}(t) \tag{26}$$

where $\varepsilon_q(t)$ is the error for state q, $q_{aiding}(t)$ is the state history obtained from the MLC aiding and $q_{nominal}(t)$ is the nominal state history. The position and velocity errors are obtained from

$$\varepsilon_{pos} = \sqrt{\left(\varepsilon_{x^e}\right)^2 + \left(\varepsilon_{y^e}\right)^2 + \left(\varepsilon_{z^e}\right)^2} \tag{27}$$

$$\varepsilon_{vel} = \sqrt{\left(\varepsilon_{vx^e}\right)^2 + \left(\varepsilon_{vy^e}\right)^2 + \left(\varepsilon_{vz^e}\right)^2} \tag{28}$$

where $\begin{bmatrix} x^e & y^e & z^e \end{bmatrix}$ is the position vector and the e-frame and $\begin{bmatrix} vx^e & vy^e & vz^e \end{bmatrix}$ is the velocity vector in the e-frame. These error measures are used to present the results in all of the following figures. For the nominal trajectory we used the GPS/INS combined solution which was produced with full-GPS availability throughout all three experiments. For the MLC approach, the satellites assumed to be in view were randomly drawn from the set of satellites that were actually in view at each instant. Monte-Carlo simulations were made where in each run different satellites in view were drawn. The same INS trajectory was used in all runs.

We begin with evaluation of the MLC approach and then compare it to the TC approach. The comparison, as a function of the number of satellites in view, is made for two GPS outage periods: 15 and 30 seconds. We then discuss how the MLC approach is further improved by selecting the FGPS satellites with the ARTDOP strategy and incorporating vehicle constraints.

5.1 MLC Applications

Figures 3-4, presents a comparison between the MLC approach (green line) and the standalone INS (red line) for two of the trajectories. The comparison is made for both position and velocity errors as a function of the number of satellites in view. As can be seen, position and velocity errors of the MLC approach are smaller relative to the standalone INS, regardless to the number of satellites in view. When three satellites are in-view a reduction of up to 75% in position error was obtained and up to 90% in the velocity error. The results are similar with all three trajectories. Considering their different characteristics, this indicates that the proposed enhancement is not scene dependent. For two satellites the error reduction is up to 25% in velocity and up to 50% in position. When only one satellite was in view the MLC approach provided only modest improvement to the standalone INS solution. This situation is circumvented, as we will show later, when introducing vehicle constraints into the solution.

Comparing the MLC to the TC approach, the figures show that both approaches improve the standalone INS performance, in particular when three satellites are in view. Also, in that case the TC approach has better performance than the MLC approach, obtaining up to 90% reduction relative to the standalone INS and up to 25% relative to the MLC approach in the position error. When two or one satellites were in view the MLC approach performed better obtaining errors that are up to 10% smaller compared to those of the TC. These results show that the proposed MLC integration has a comparable effect in reducing the navigation error as the TC, which is a further indication to its appeal.

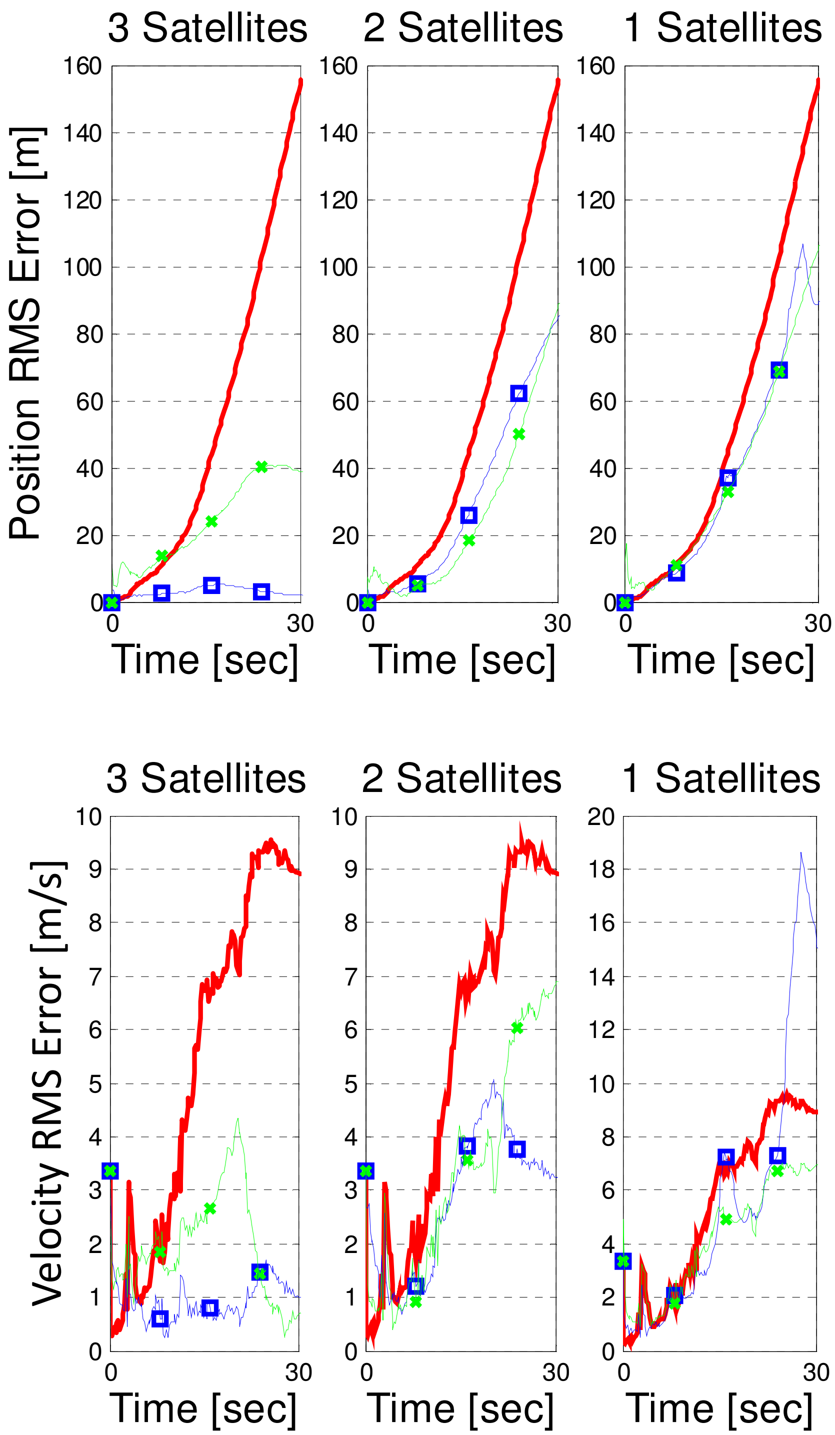

Figure 3: Trajectory 1 position and velocity errors for MLC and TC approaches, standalone INS: red line, MLC: green line, TC: blue line.

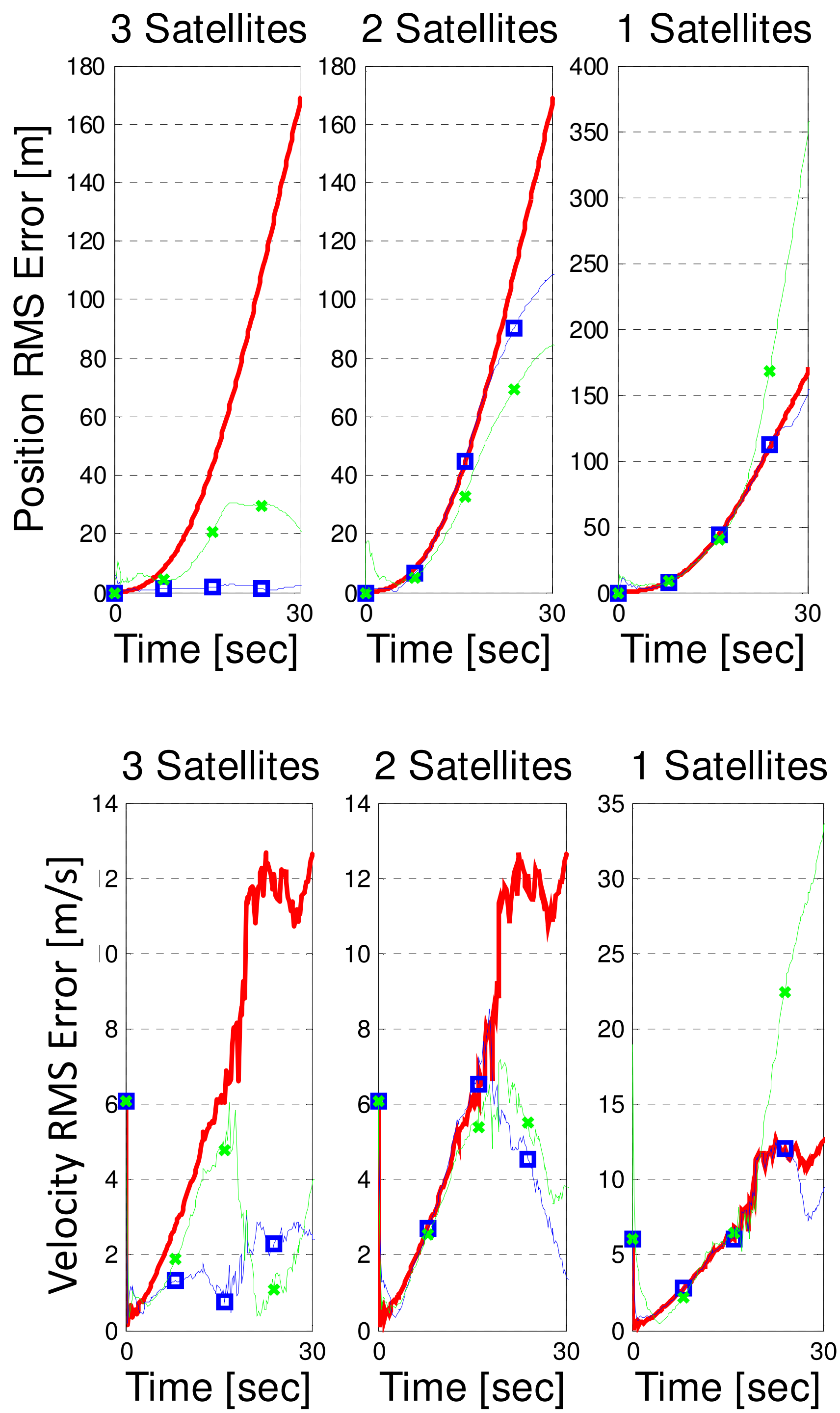

Figure 4: Trajectory 2 position and velocity errors for MLC and TC approaches, standalone INS: red line, MLC: green line, TC: blue line.

The mean position and velocity errors of MLC and TC approaches, after 15 and 30 seconds of GPS outage, and depending on the number of satellites in view are summarized in Table 1 and 2, respectively for the three trajectories. In the tables, the best results obtained with each trajectory for each number of satellites in view, is highlighted.

Table 1: Position Error of TC and MLC approaches

	Position Error (mean) [m] after 15 [sec]						Position Error (mean) [m] after 30 [sec]					
	3 Sats.		2 Sats.		1 Sats.		3 Sats.		2 Sats.		1 Sats.	
	MLC	TC	MLC	TC	MLC	TC	MLC	TC	MLC	TC	MLC	TC
Trajectory 1	12.6	2.57	6.11	7.25	12.3	9.90	23.5	3.05	25.8	30.7	37.6	37.8
Trajectory 2	6.42	1.32	10.1	10.6	12.7	12.2	16.9	1.62	35.8	45.5	82.2	53.1
Trajectory 3	5.97	2.14	7.00	7.64	7.86	8.21	10.4	2.24	18.2	23.7	36.4	40.8

Table 2: Velocity Error of TC and MLC approaches

	Velocity Error (mean) [m/s] after 15 [sec]						Velocity Error (mean) [m/s] after 30 [sec]					
	3 Sats.		2 Sats.		1 Sats.		3 Sats.		2 Sats.		1 Sats.	
	MLC	TC	MLC	TC	MLC	TC	MLC	TC	MLC	TC	MLC	TC
Trajectory 1	1.84	0.89	1.72	1.77	2.05	1.95	1.98	0.96	3.34	2.84	4.22	5.65
Trajectory 2	2.26	1.26	2.63	2.99	3.36	2.96	2.36	1.71	3.97	3.88	10.2	6.22
Trajectory 3	1.16	0.50	0.97	1.45	1.29	1.68	1.51	0.65	2.07	2.28	3.67	4.65

In summary, the results above indicate the following outcomes: 1) both MLC and TC schemes operate well when less than four satellites are in view; 2) when three satellites are in view the TC approach outperforms the MLC with respect to both the position and the velocity errors; 3) when two satellites are in view, the MLC shows better performance than the TC approach both in position and velocity errors; 4) when one satellite is in view the performance of the two approaches was similar.

We next evaluate the potential improvements to the proposed approach by FGPS satellites selection with the ARTDOP strategy and incorporation of vehicle constraints. Evaluation of the ARTDOP sole contribution compared to the DOP approach shows relatively modest improvement (~5%). This can be expected since the DOP solution produced appropriate satellites positions in most cases. Evaluating the contribution of the vehicle constraints, two types of fusions are considered: 1) MLC with a constant height constraint and FGPS satellite selection using the ARTDOP methodology; 2) MLC with constant height body velocity constraints, and ARTDOP based FGPS satellite selection. In Figure 5 and Tables 3 and 4 we present the results obtained with the second trajectory. The results obtained for the other two trajectories are similar.

Table 3: Position Error for several fusion approaches

	Position Error (mean) [m] after 15 [sec]			Position Error (mean) [m] after 30 [sec]		
	3 Sats.	2 Sats.	1 Sats.	3 Sats.	2 Sats.	1 Sats.
Standalone INS	11.0	11.0	11.0	54.4	54.4	54.4
MLC	6.41	10.1	12.7	16.9	35.8	82.2
MLC+ARTDOP	6.54	10.0	12.4	17.2	34.4	82.2
MLC+HC+ARTDOP	9.41	6.62	5.00	16.1	29.6	23.8
MLC+HC+VC+ARTDOP	10.4	7.54	4.81	14.4	29.0	23.1

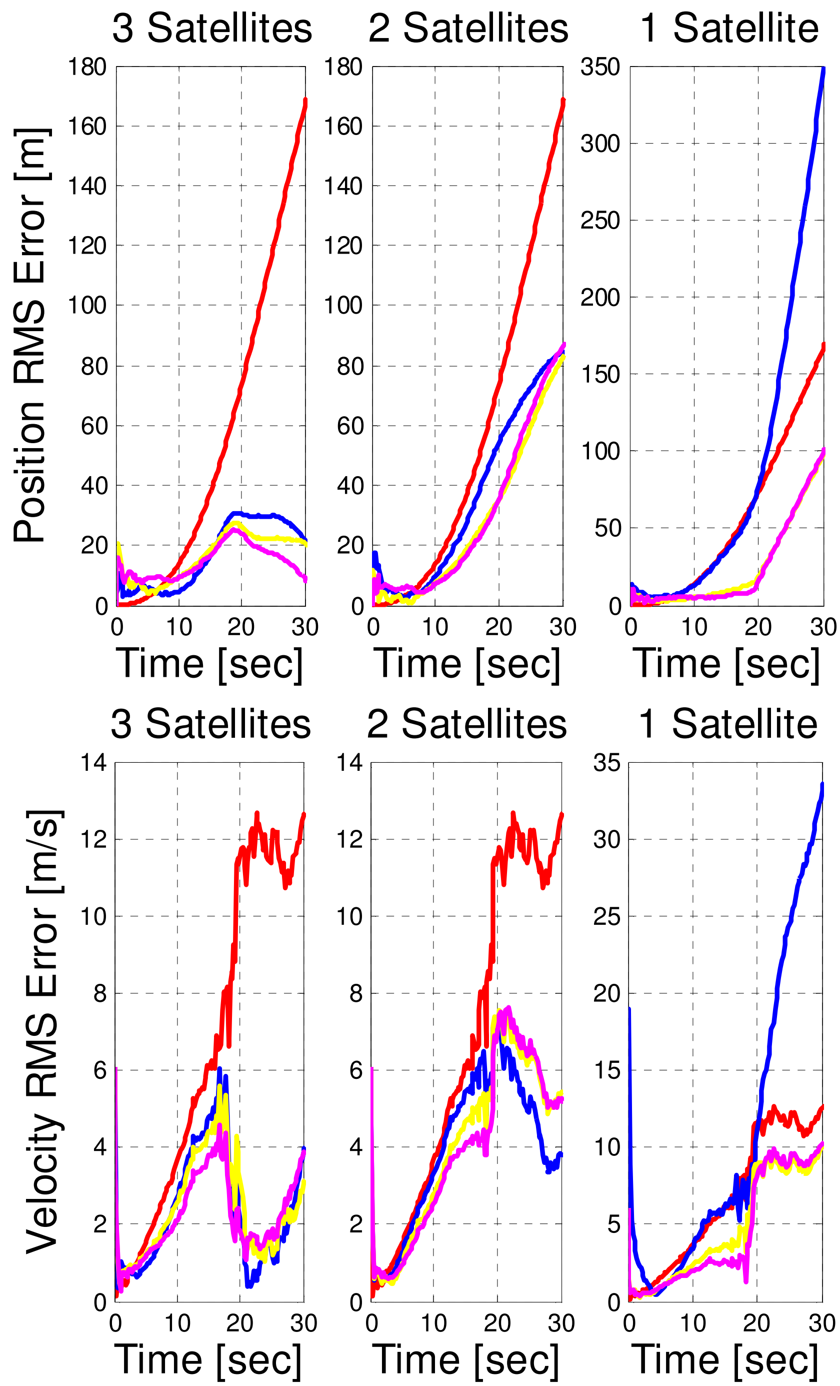

Figure 5: Position and velocity errors for MLC with vehicle constraints and ARTDOP, standalone INS: red line, MLC approach: blue line, MLC+HC+ARTDOP: yellow line, MLC+HC+VC+ARTDOP: magenta line.

Table 4: Velocity Error for several fusion approaches

	Velocity Error (mean) [m/s] after 15 [sec]			Velocity Error (mean) [m/s] after 30 [sec]		
	3 Sats.	2 Sats.	1 Sats.	3 Sats.	2 Sats.	1 Sats.
Standalone INS	2.82	2.82	2.82	6.61	6.61	6.61
MLC	2.26	2.63	3.36	2.36	3.97	10.2
MLC+ARTDOP	2.21	2.54	3.32	2.41	3.78	10.1
MLC+HC+ ARTDOP	2.21	2.26	1.85	2.41	4.07	4.72
MLC+HC+VC+ARTDOP	1.93	2.11	1.57	2.20	3.93	4.41

The results show that this integration scheme has a significant effect on reducing the navigation error. The results reveal that 1) fusion of the MLC approach with constant height and body velocity constraints and selection the FGPS satellites with the ARTDOP methodology had improved the performance of the standard MLC approach; 2) a slight improvement of ~5% was achieved when three satellites were in view and a 50% reduction was obtained when one satellite was in view for the position error, 3) The velocity errors of the MLC approach were improved regardless to the number of satellites in view. In particular when one satellite was in view an improvement of about 50% was obtained relative to the standalone INS.

6. Conclusions

This paper proposed the MLC integration approach for cases in which LC integration is not possible since there are less than 4 satellites in view. To apply the proposed approach, two strategies for positioning the FGPS satellites were proposed: 1) The DOP approach using the positions of real satellites that are not currently in view, and 2) The ARTDOP approach which uses artificial GPS satellites. To further enhance the performance of the proposed approach, two vehicle constraints were fused with each strategy.

Results show that the proposed introduction of FGPS satellites offers a viable option to a loose coupling based integration. They have also shown that fusion of the constant height and body velocity constraints with the selection of the FGPS satellites considerably improved the performance of the standalone INS both in the position and velocity errors regardless of the examined trajectory and number of satellites in view.

The proposed model requires no hardware change, but only the addition of the MLC algorithm to the software. Thus, with this small modification, the loosely coupled approach can be implemented even with one available satellite and yield reduced position and velocity errors.

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