Estimation of an Integrated Driving Behavior Model

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Abstract

This paper presents the methodology and results of estimation of an integrated driving behavior model that attempts to integrate various driving behaviors. The model explains lane changing and acceleration decisions jointly and so, captures inter-dependencies between these behaviors and represents drivers' planning capabilities. It introduces new models that capture drivers' choice of a target gap that they intend to use in order to change lanes, and acceleration models that capture drivers' behavior to facilitate the completion of a desired lane change using the target gap.

The parameters of all components of the model are estimated simultaneously with the maximum likelihood method and using detailed vehicle trajectory data collected in a freeway section in Arlington, VA. The estimation results are presented and discussed in detail.

1 Introduction

Driving behavior models are fundamental to the understanding of traffic flow phenomena and form the basis for microscopic traffic simulation models. A vast body of literature (see reviews in Gerlough and Huber 1975, Leutzbach 1988, Brackstone and McDonald 1999, Rothery 2001, Hoogendoorn and Bovy 2001, Toledo 2007, among others) deals with the specification and estimation of these models, and in particular acceleration and lane changing models. However, these two behaviors are commonly modeled and implemented independently of each other.

Toledo et al. (2007b) demonstrated the potential shortcomings of the independent modeling approach and presented an integrated framework to jointly model acceleration and lane changing behaviors that can represent drivers' planning capabilities. The structure of the integrated model is shown in Figure 1. It assumes that drivers develop short-term plans to accomplish short-term goals. The short-term goal is defined by a target lane, which is the lane the driver perceives as best to be in among the *Current*, *Right* or *Left* lanes. In the case that either the right lane or the left lane are chosen, the driver evaluates the adjacent gap in the target lane and decides whether it is acceptable or not. If the gap is accepted (*Change right* or *Change left*), the lane change is immediately executed. If the available gap is rejected (*No change*), the driver develops a short-term plan by choosing a target gap, which will be used to complete the desired lane change (*Gap* R_1 to *Gap* R_K or *Gap* L_1 to *GAP* L_M). The acceleration the driver applies depends on the short-term goal and plan – It may be affected by the leader in the current lane or in the new lane the vehicle is changing to, or it can be adapted to facilitate the short-term plan. (i.e. the driver positions the vehicle such that the target gap will be acceptable). The target lane and target gap are both unobservable. Only the driver's actions (lane changes and accelerations) are observed. In the figure latent choices are shown as ovals. Observed choices are shown as rectangles.

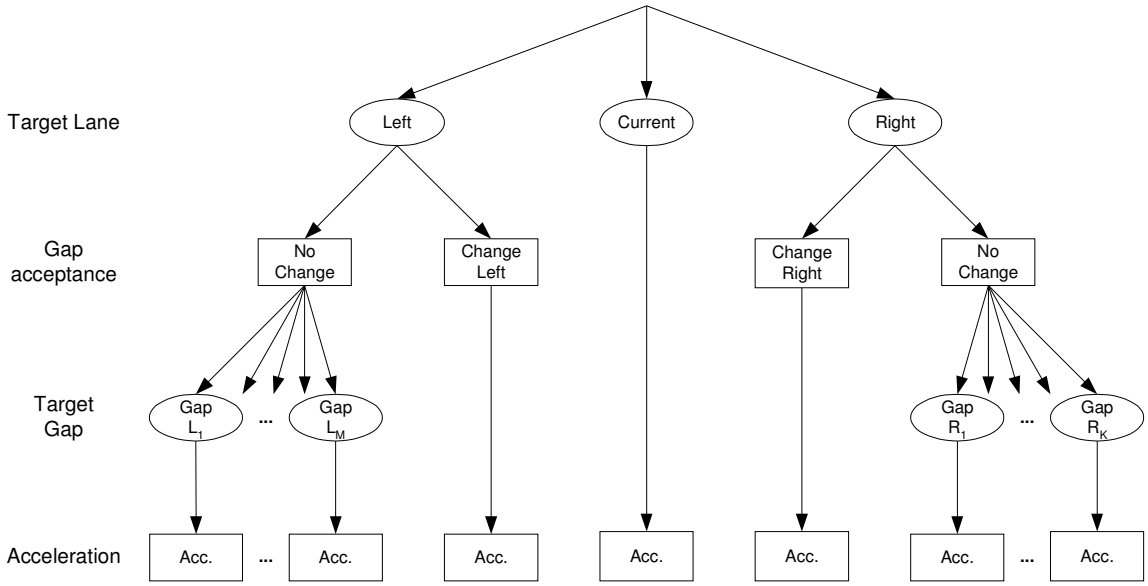

Figure 1 Structure of the integrated driving behavior model

In this paper, a methodology for the estimation of the integrated model and results obtained from the application of the methodology using a dataset of vehicle trajectories are presented. The rest of this paper is organized as follows: the next section describes the data requirements and the set of trajectory dataset used to estimate the parameters of the model. The model likelihood function is formulated in Section 3. Estimation results are presented in section 4. Finally, conclusions are presented in section 5.

2 Data for Model Estimation

Estimation of the integrated driving behavior model requires detailed trajectory data, which consists of observations of the positions of vehicles at discrete points in time, and other variables that may be derived from these positions (e.g. speeds, accelerations, lane changes, headways and lengths of gaps in traffic). The estimation dataset needs to cover the important

variables that may be used to explain driving decisions. These variables may be broadly classified into four categories:

1. Neighborhood variables, which describe the subject vehicle and its relations with surrounding vehicles (e.g. relative speeds and spacing with respect to vehicles in front of and behind the subject and in adjacent lanes, and the presence of heavy vehicles) and capture traffic conditions in the extended environment of the vehicle (e.g. densities and average speeds and their lane distributions).
2. Trip plan variables, which capture the effect of drivers' desire to follow their paths and to adhere to the trip schedule on their driving (e.g. lane selection, desired speeds). Relevant variables may include distances to points where drivers must be in specific lanes to follow their path, the number of lane changes required to be in the correct lanes, whether the driver is ahead or behind schedule and so on.
3. Network knowledge and experience variables, which capture preferences that are based on drivers' familiarity and understanding of the transportation system. For example, drivers may prefer the left freeway lanes to minimize delays from weaving traffic or avoid following a bus that makes service stops.
4. Driving style and capability variables, which capture the individual characteristics of the driver (e.g. aggressiveness and reaction times) and of the vehicle (e.g. speed and acceleration capabilities). These variables are often not observed. However, their effects may be captured by appropriate model specifications, such as introduction of individual-specific effects to capture correlations among the various decisions a driver makes.

The dataset used in this study was collected in a section of I-395 Southbound in Arlington VA, shown in Figure 2 (FHWA 1985). The four-lane highway section is 997 meter long. The dataset includes 15632 observations from 442 vehicles at 1 second time resolution. 76% of the vehicles remain on the freeway at the downstream end of the section, 8% take the first off-ramp and 16% take the second one. Vehicles' speeds range from 0.4 to 25.0 m/sec. with a mean of 15.6 m/sec. Densities range from

14.2 to 55.0 veh/km/lane with a mean of 31.4 veh/km/lane. Acceleration observations vary from -3.97 to 3.99 m/sec^2. The level of service in the section is D-E (HCM 2000). The discrete observations were smoothed and continuous trajectory functions were estimated using the local regression method (see e.g. Toledo et al. 2007a). Further details on the data collection and reduction methods are presented in FHWA (1985). Analysis of the data is presented in Toledo (2003).

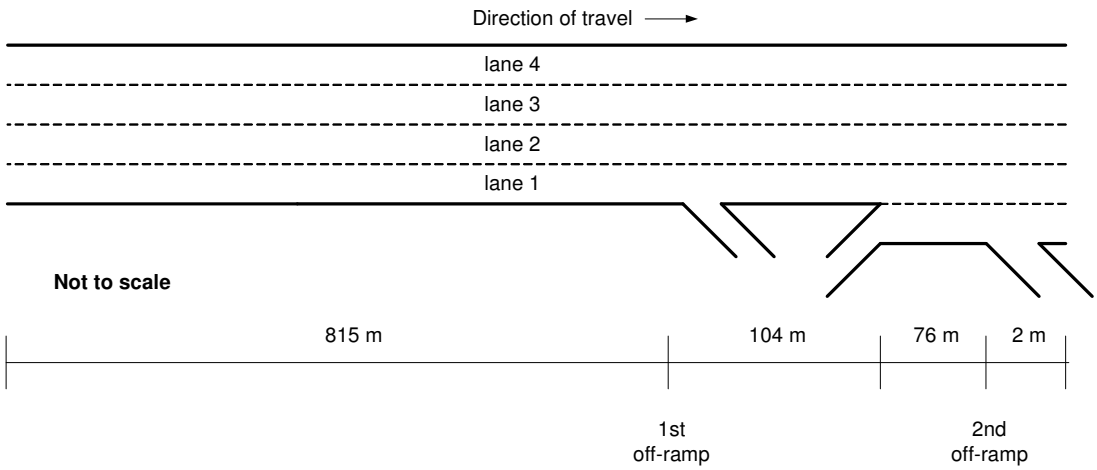

Figure 2 Data collection site

3 Likelihood function

As discussed above, the path plan is an important factor that affects driving behavior through variables such as the distance to an off-ramp the driver intends to use. However, trajectory datasets are collected only on a limited section of the road and therefore, the path plans of drivers that remain on the freeway at the downstream end of the section are not observed. In order to capture the effect of these variables, a distribution of the distances from the downstream end of the section to the exit points for these vehicles is introduced. The parameters of this distribution are estimated jointly with the other parameters of the model. A discrete distribution of the exit distances is used, which exploits information on the locations of off-ramps downstream of the section. The alternatives considered are the first, second

and subsequent off-ramps beyond the downstream end of the section. The probability mass function of distance to the exit off-ramps, is given by:

$$p(d_n) = \begin{cases} \pi_1 & d^1 \text{ (first downstream exit)} \\ \pi_2 & d^2 \text{ (second downstream exit)} \\ 1-\pi_1-\pi_2 & d^3 \text{ (otherwise)} \end{cases} \quad (1)$$

π_1 and π_2 are the parameters corresponding to the probabilities that a vehicle will exit at the first and second downstream off-ramp, respectively. d^1, d^2 and d^3 are the distances beyond the downstream end of the section to the first, second and subsequent exits, respectively.

The first and second exit distances (d^1=300m and d^2=550m) are measured directly. For the subsequent exits an infinite distance is used ($d^3 = \infty$), which corresponds to the assumption that the driver ignores this consideration in lane choices in the study area. In addition, driver-specific latent variables are introduced to capture correlations among the decisions made by the same driver. These variables, the individual-specific error term υ_n, reaction times (τ_n) and time headway thresholds (h_n^*), are randomly distributed in the population. The joint probability density of a combination of target lane (*TL*), lane action (*l*), target gap (*TG*) and acceleration (*a*) for driver *n* at time *t*, conditional on the individual-specific variables (d_n, υ_n, τ_n and h_n^*) is given by:

$$\begin{aligned} f_n\left(TL(t),l(t),TG(t),a(t)|d_n,\upsilon_n,\tau_n,h_n^*\right) &= P_n\left(TL(t)\,|\,d_n,\upsilon_n\right)P_n\left(l(t)\,|\,TL(t),\upsilon_n\right)\cdot \\ &\cdot P_n\left(TG(t)\,|\,TL(t),l(t),\upsilon_n\right)f_n\left(a(t)\,|\,TL(t),l(t),TG(t),\upsilon_n,\tau_n,h_n^*\right) \end{aligned} \quad (2)$$

$P_n\left(TL(t)\,|\cdot\right)$, $P_n\left(l(t)\,|\cdot\right)$, $P_n\left(TG(t)\,|\cdot\right)$ and $f_n\left(a(t)\,|\cdot\right)$ are the target lane, gap acceptance, target gap and acceleration probability density functions, respectively. The detailed specifications of these functions are described in Toledo et al. (2007).

Only the lane changing and acceleration decisions are observed. Their joint probability density function is obtained as the marginal of Equation (2):

$$f_n\left(l(t),a(t)|d_n,\upsilon_n,\tau_n,h_n^*\right)=\sum_{TL(t)}\sum_{TG(t)} f_n\left(TL(t),l(t),TG(t),a(t)|d_n,\upsilon_n,\tau_n,h_n^*\right) \quad (3)$$

The behavior of each driver n is observed over a sequence of T_n consecutive time intervals. Assuming that conditional on the driver-specific latent variables these decisions are independent, the joint probability of the sequence of observations is given by:

$$f_n\left(\mathbf{l},\mathbf{a}|d_n,\upsilon_n,\tau_n,h_n^*\right)=\prod_{t=1}^{T_n} f_n\left(l(t),a(t)|d_n,\upsilon_n,\tau_n,h_n^*\right) \quad (4)$$

l and **a** are the sequences of lane changing decisions and accelerations, respectively.

The unconditional individual likelihood function is obtained by integration of the conditional probability over the distributions of the individual specific variables:

$$L_n=\int_{\upsilon}\int_{\tau}\int_{h^*}\sum_{d} f_n\left(\mathbf{l},\mathbf{a}\,|\,d_n,\upsilon_n,\tau_n,h_n^*\right)p\left(d\right)f\left(h^*\right)f\left(\tau\right)f\left(\upsilon\right)dh^*d\tau d\upsilon \quad (5)$$

$p\left(d\right)$ is given by Equation (1). $f\left(h^*\right)$ and $f\left(\tau\right)$ are described in Toledo et al. (2007). $f\left(\upsilon\right)$ is the standard normal probability density function.

Assuming that observations of different drivers are independent, the log-likelihood function for all N individuals observed is given by:

$$L=\sum_{n=1}^{N}\ln\left(L_n\right) \quad (6)$$

A limitation of the formulation above is that it does not explicitly account for state-dependencies in the driving process since it assumes that drivers choose a short-term goal and short-term plan at every time step without regards to their earlier choices and therefore that state-dependencies are captured by the explanatory variables. This resulting model is significantly more efficient computationally compared to a state-dependencies model,

which would require explicit enumeration of all the possible sequences of short-term goals and short-term plans. Furthermore, the values of explanatory variables that are derived from the positions and speeds of the subject vehicle and surrounding vehicles depend on earlier decisions the driver made (e.g. the vehicle speed and position depend on past accelerations) and so their inclusion in the model may capture the effects of previous decisions. Nevertheless, formulations that allow efficient integration of Markovian processes in choice models have recently been presented by Choudhury et al. (2007). Their incorporation in the models presented here may lead to further improvements.

4 Estimation results

All components of the model were estimated jointly using the maximum likelihood function described above. However, in order to simplify the presentation, estimation results for the various components of the model are presented separately. Most of the parameter's estimates presented are statistically significant. In some cases structural parameters (such as α and EMU) and variables whose impact was as expected were retained in the model even when their significance levels were lower, in particular in the gap choice and gap acceleration models that are explored here for the first time.

4.1 The target lane model

This model explains the choice of the short-term goal, which is defined in terms of a target lane the driver perceives as best to be in. The alternatives are the current lane (*CL*), the right lane (*RL*) or the left lane (*LL*). The utilities of the various lanes to driver *n* at time *t* are given by:

$$U_n^{lane\,i}(t) = X_n^{lane\,i}(t)\beta^{lane\,i} + \gamma^{lane\,i} EMU_n^{lane\,i}(t) + \alpha^{lane\,i}\upsilon_n + \varepsilon_n^{lane\,i}(t) \qquad (7)$$

$lane\ i \in TL = \{CL, RL, LL\}$. $X_n^{lane\,i}(t)$ are vectors of explanatory variables affecting the utility of lane *i*. $\beta^{lane\,i}$ are the corresponding vectors of

parameters. $EMU_n^{lane\,i}(t)$ are the expected maximum lower level utilities of target lanes that trigger a lane change. $\gamma^{lane\,i}$ are the parameters of the expected maximum utilities. $\varepsilon_n^{lane\,i}(t)$ are the random terms associated with the lane utilities. $\alpha^{lane\,i}$ are the parameters of υ_n.

Table 1 Estimation results for the target lane model

Variable	Parameter value	t-statistic
Current Lane (CL) constant	2.128	2.68
Right Lane (RL) constant	-0.369	-1.28
Path plan impact, 1 lane change required	-2.269	-5.57
Path plan impact, 2 lane changes required	-4.466	-7.18
Path plan impact, 3 lane changes required	-7.265	-8.34
θ^{MLC}	-0.358	-2.74
Next exit impact dummy, 1 lane change required	-1.264	-2.92
Next exit impact dummy, each additional lane change	-0.252	-1.36
Front vehicle speed, m/sec.	0.0745	1.78
Front vehicle spacing, m.	0.0225	3.68
Heavy neighbor dummy	-0.218	-0.93
Tailgate dummy	-3.793	-1.83
Lane density, veh/km/lane	-0.0018	-1.45
Right-most lane dummy	-1.039	-3.85
Gap acceptance expected maximum utility, *EMU*	0.0052	0.41
π_1	0.0063	0.57
π_2	0.0406	1.16
α^{CL}	0.539	5.07
α^{RL}	1.035	5.15

Estimation results of the target lane model are presented in Table 1. The estimated model constants indicate that everything else being equal drivers strongly prefer to stay in the current lane. The impact of the driver's path plan is captured by a group of variables, which combine a function of the distance to the point where the driver needs to be in a specific lane in order to take an off-ramp and the number of lane changes required to be in the correct lane. For the estimation dataset, three variables are defined:

$$path\_plan\_impact\_j_n^{lane\ i}(t) = \left[d_n^{exit}(t)\right]^{\theta^{MLC}} \delta_n^{j\,i}(t) \qquad \begin{array}{c} i \in \{CL, RL, LL\} \\ j \in \{1,2,3\} \end{array} \tag{8}$$

$d_n^{exit}(t)$ is the distance in kilometers from the position of vehicle n at time t to the point where it needs to take the exit. θ^{MLC} is a parameter to be estimated. $\delta_n^{j,i}(t)$ are indicators of the number of lane changes required to follow the path:

$$\delta_n^{j,i}(t) = \begin{cases} 1 & j \text{ lane changes are required from lane } i \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

As expected, the utility of a lane decreases with the number of lane changes the driver needs to make from it. This effect is magnified as the distance to the off-ramp decreases (θ^{MLC} = -0.358). Figure 3 demonstrates the impact of required lane changes on the probability targeting the right lane as a function of the distance from the off-ramp.

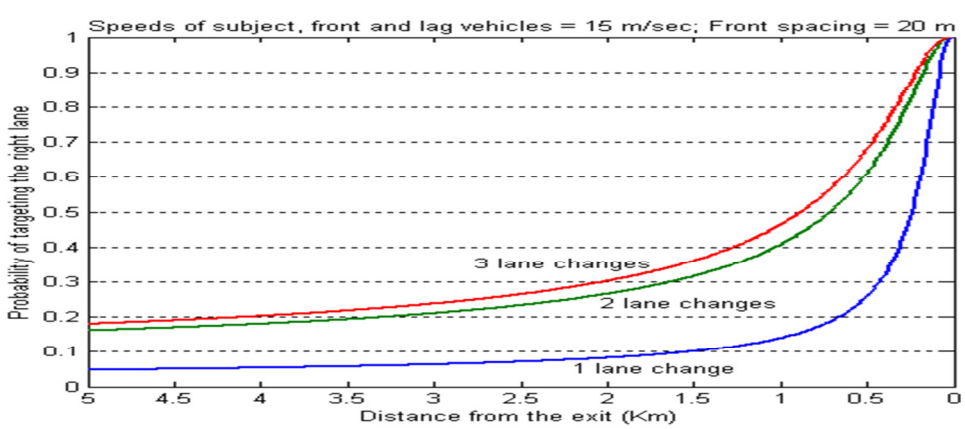

Figure 3 Impact of required lane changes on the probability of targeting the right lane

Drivers who intend to exit the freeway at the next off-ramp may be more likely to pre-position themselves in the correct lane compared to drivers who use subsequent exits. Explanatory variables that capture this behavior are generated by interaction of a next-exit dummy variable with the number of lane changes required:

$$next\_exit\_impact\_1_n^{lane\ i}(t) = \delta_n^{next\ exit}(t)\delta_n^{1,i}(t) \quad i = CL, RL, LL \tag{10}$$

$$next\_exit\_impact\_2+_n^{lane\ i}(t) = \delta_n^{next\ exit}(t)\delta_n^{add,i}(t) \quad i = CL, RL, LL \tag{11}$$

$\delta_n^{1,i}(t)$ is defined in Equation (9). The indicator variables $\delta_n^{next\ exit}(t)$ and $\delta_n^{add,i}(t)$ are given by:

$$\delta_n^{next\ exit}(t) = \begin{cases} 1 & \textit{the next off-ramp is used} \\ 0 & \textit{otherwise} \end{cases} \tag{12}$$

$$\delta_n^{add,i}(t) = \begin{cases} 2 & \delta_n^{3,i}(t) = 1 \\ 1 & \delta_n^{2,i}(t) = 1 \\ 0 & \textit{otherwise} \end{cases} \qquad i = CL, RL, LL \tag{13}$$

The estimated coefficients for these variables are negative and the utility of a lane decreases with the number of lane changes required. The marginal disutility associated with needing one lane change to take the next off-ramp is larger than that of any additional lane changes. The parameter of the additional lane changes is also not highly significant (t-value -1.36). This implies that drivers perceive being in the wrong lane as a more significant factor compared to the number of lane changes that are required.

The variables that capture driving conditions in the subjects' neighborhood include the speed of the vehicle in front of the subject and the spacing between them, densities in the various lanes, presence of heavy vehicles and tailgaters and the expected maximum utilities of the available gaps in the lanes to the right and to the left of the subject vehicle. Overall, these variables are less important in the utility function compared to those related to the path plan and their statistical significance is also generally lower. The speed and the spacing of the front vehicle variable, which only appear in the

utility of the current lane, capture the disturbance this vehicle poses to the subject. The utility of the current lane increases as the values of both these variables increase. Another variable that captures driving conditions in the subject's neighborhood is the presence of heavy vehicles. This dummy variable is defined separately for each candidate lane:

$$\delta_n^{heavy\ neighbor,\ i}(t) = \begin{cases} 1 & \textit{lead and/or lag in lane i is heavy} \\ 0 & \textit{otherwise} \end{cases} \qquad i = CL, RL, LL \tag{14}$$

A vehicle is defined as heavy if its length exceeds 9.14 meters (30 feet). The parameter of this variable had the expected sign but was not significant in the model. It was kept in the model because of its usefulness in implementation in traffic simulation models that incorporate various vehicle types. The utility of a lane decreases if there are neighboring heavy vehicles in that lane, which captures drivers' preference to avoid interacting with heavy vehicles. The tailgating dummy variable captures drivers' tendency to move out of their current lane if they are being tailgated. Tailgating is assumed if a vehicle is close to the vehicle in front of it (the subject vehicle) when traffic conditions permit a longer headway (i.e. free-flow conditions apply). Mathematically, the tailgate dummy variable is defined by:

$$\delta_n^{tailgate}(t) = \begin{cases} 1 & \textit{gap behind} \leq \textit{10m and level of service is A, B or C} \\ 0 & \textit{otherwise} \end{cases} \tag{15}$$

Levels of service definitions are based on densities (HCM 2000). The estimated coefficient of the tailgate dummy is large and negative, which implies a strong preference to avoid these situations. This result is comparable with those of Ahmed (1999), who also found tailgating to be an important explanatory variable. The lane density variable captures conditions in an extended neighborhood. The utility of a lane decreases with higher densities. The right-most lane dummy variable takes a value of 1 if the lane is the right-most and 0 otherwise. It captures drivers' preference to avoid the merging and weaving that occurs in this lane.

The expected maximum utilities (*EMU*) of the available gaps in the right lane and in the left lane capture the impact of gap acceptance decisions on the target lane choice. The values of these variables increase with the probability that the subject vehicle will be able to accept the available gaps in these lanes if they are chosen as the target lane. The estimated coefficient of this variable is positive, which indicates that drivers are more likely to target a lane change when the completion of the lane change is easier. The heterogeneity coefficients, α^{CL} and α^{RL}, capture the effects of the individual-specific error term υ_n on the target lane choice, thus accounting for correlations between observations of the same individual due to unobserved characteristics of the driver/vehicle. Both estimated parameters are positive and so, υ_n can be interpreted as associated with drivers' timidity. A timid driver (i.e. $\upsilon_n > 0$) is more likely to choose the right lane and the current lane over the left lane compared to a more aggressive driver.

4.2 The gap acceptance model

The gap acceptance model explains the decision whether or not to change to the target lane immediately. It assumes that both the lead gap and the lag gap must be acceptable in order for the vehicle to change lanes. The lead and lag gaps are the clear spacing between the subject vehicle and the lead and lag vehicles in the target lane, respectively. Available gaps are accepted only if they are greater than the corresponding critical gaps, which are modeled as random variables whose means are functions of explanatory variables. In order to ensure positive critical gaps, they are assumed to follow a lognormal distribution:

$$\ln\left(G_n^{gap\,i\,TL,cr}(t)\right) = X_n^{gap\,i\,TL}(t)\beta^{gap\,i} + \gamma^{gap\,i} EMU_n^{gap\,i\,TL}(t) + \alpha^{gap\,i}\upsilon_n + \varepsilon_n^{gap\,i}(t) \qquad (16)$$

$gap\ i \in \{lead, lag\}$. $X_n^{gap\,i\,TL}(t)$ are vectors of explanatory variables affecting the critical gaps. $\beta^{gap\,i}$ are the corresponding vectors of parameters. $\varepsilon_n^{gap\,i}(t) \sim N\left(0, \sigma^2_{gap\,i}\right)$ are error terms. $EMU_n^{gap\,i\,TL}(t)$ and $\gamma^{gap\,i}$ are the expected

maximum lower level utilities and the corresponding parameters, respectively. $\alpha^{gap\,i}$ are the parameters of the individual-specific random term υ_n.

Critical gaps estimation results are presented in Table 2. Both the lead critical gap and the lag critical gap depend on the subject relative speed with respect to the corresponding vehicles. The relative speed with respect to a neighboring vehicle is defined as the speed of that vehicle less the subject speed. The lead critical gap is larger when the subject is faster relative to the lead vehicle and decreases when the relative speed increases. When the lead vehicle is faster than the subject, the critical gap reduces to almost zero. This result suggests that drivers perceive very little risk from the lead vehicle when it is getting away from them. Inversely, the lag critical gap increases when the speed of the lag vehicle is higher relative to the subject. Unlike, the lead critical gap, the lag gap does not diminish when the subject is faster. A possible explanation may be that drivers keep a minimum critical lag gap as a safety buffer because their view of this gap is not as clear.

The *EMU* variables capture the effects of available gaps on critical gaps - larger values occur when available gaps are larger. Both the lead and lag critical gaps increase with the target gap *EMU*, which suggests that drivers adapt the risk they are willing to take to conditions: when available gaps are smaller drivers tend to require smaller critical gaps compared to the case where available gaps are larger. The effect of this variable is stronger in the lag critical gap relative to the lead critical gap. This may again be explained by the higher uncertainty and extra caution associated with the lag gap. Median lead and lag critical gaps, as a function of the relative speeds and *EMU* are presented in Figure 4. Estimated coefficients of υ_n are positive for both the lead and the lag critical gaps. These values are consistent with the

interpretation that υ_n characterizes timid drivers that require larger gaps for lane changing compared to more aggressive drivers.

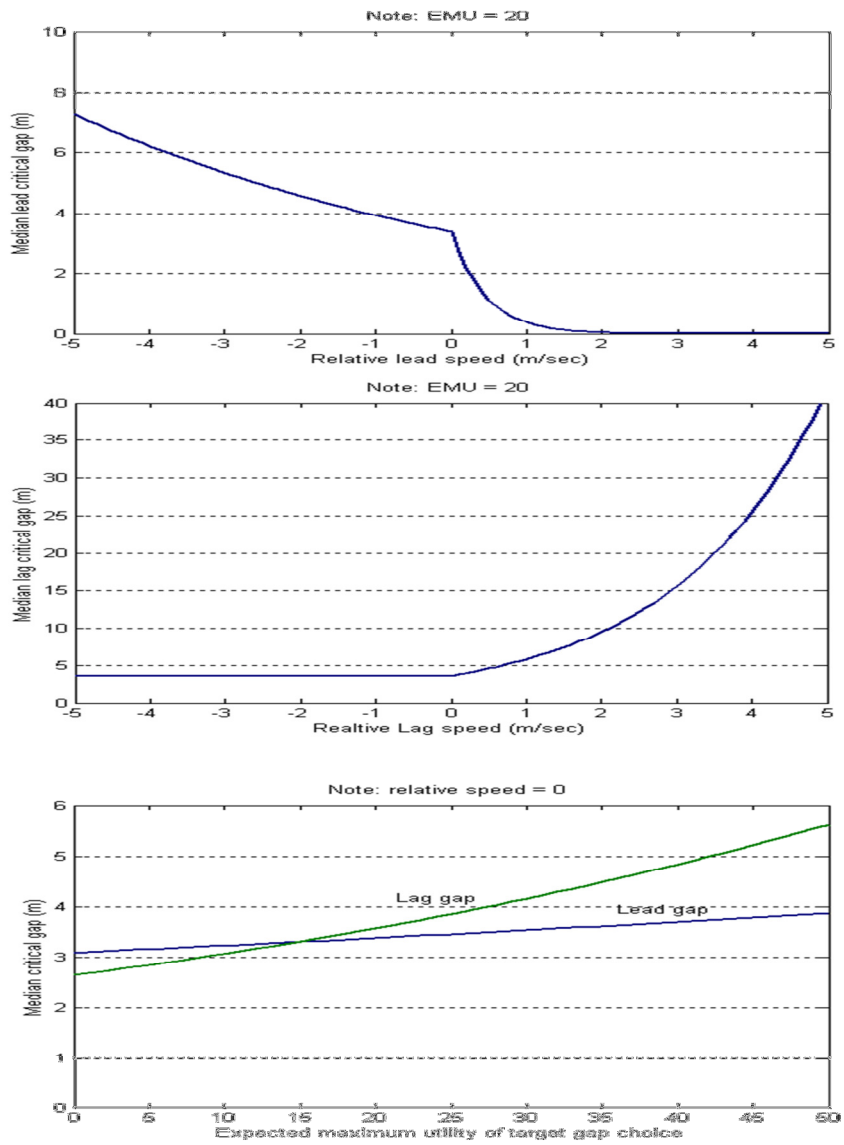

Figure 4 Impact of relative speeds and *EMU* on median lead and lag critical gaps

Table 2 Estimation results for the gap acceptance model

Variable	Parameter value	t-statistic
Lead Critical Gap		
Constant	1.127	2.78
$Max\left(\Delta V_n^{lead}(t),0\right)$, m/sec.	-2.178	-0.63
$Min\left(\Delta V_n^{lead}(t),0\right)$, m/sec.	-0.153	-1.86
Target gap expected maximum utility, *EMU*	0.0045	1.29
α^{lead}	0.789	2.46
σ^{lead}	1.217	2.55
Lag Critical Gap		
Constant	0.968	4.18
$Max\left(\Delta V_n^{lag}(t),0\right)$, m/sec.	0.491	5.95
Target gap expected maximum utility	0.0152	1.65
α^{lag}	0.107	0.47
σ^{lag}	0.622	4.53

4.3 The target gap model

The target gap is the gap in traffic in the target lane that the driver plans to use to execute the desired lane change, if the adjacent gap is rejected. The choice set includes three alternatives: the adjacent gap, forward gap and backward gap. The utilities of the different target gaps are given by:

$$U_n^{gap\ i}(t) = X_n^{gap\ i}(t)\beta^{gap\ i} + \alpha^{gap\ i}\upsilon_n + \varepsilon_n^{gap\ i}(t) \tag{17}$$

$gap\,i \in \{adjacent, forward, backward\}$. $X_n^{gap\,i}(t)$ is a vector of explanatory variables affecting the utility of gap i. $\beta^{gap\,i}$ are the corresponding parameters. $\varepsilon_n^{gap\,i}(t)$ are random terms. $\alpha^{gap\,i}$ are the parameters of υ_n.

Estimation results for this model are presented in Table 3. The distance to gap variable captures the proximity of the subject to the target gap. It is defined in terms of space headways as shown in Figure 5. The distances to the forward and backward gaps are non-negative, and the distance to the adjacent gap is by definition equal to zero. The estimated coefficient of this variable is negative and large, which implies a strong preference for the adjacent gap over the alternative gaps. Furthermore, as the distance to a gap increases, the short-term plan using that gap is likely to take longer to be completed and involve more uncertainty with respect to the behavior of other vehicles. The forward gap alternative-specific constant is negative and the one for the backward gap is positive. Coupled together, these results imply that, everything else being equal, drivers tend to prefer the adjacent gap and backward gaps to the forward gap, which reflects risk-averse behavior.

The effective gap length defines the length of the gap in question that is available for the subject vehicle, taking into account the position of the front vehicle. Mathematically it is given by:

$$EG_n^{gap\ i}(t) = Min\left(\Delta X_n^{gap\ i,TL}(t), \Delta X_{n,front}^{gap\ i,TL}(t)\right) \tag{18}$$

$EG_n^{gap\ i}(t)$ is the effective length of gap i. $\Delta X_n^{gap\ i,TL}(t)$ and $\Delta X_{n,front}^{gap\ i,TL}(t)$ are the length of the gap and the spacing between the vehicle at the rear of the gap and the front vehicle, respectively, as illustrated for the forward gap in Figure 5. As expected, the utility of a gap increases with its effective length.

Table 3 Estimation results for the target gap model

Variable	Parameter value	t-statistic
Forward gap constant	-0.837	-0.50
Backward gap constant	0.913	4.40
Distance to gap, m.	-2.393	-7.98
Effective gap length, m.	0.816	2.20
Gap rate of change, m/sec.	-1.218	-4.00
Front vehicle dummy	-1.662	-1.53
α^{bck}	0.239	0.81
α^{adj}	0.675	0.95

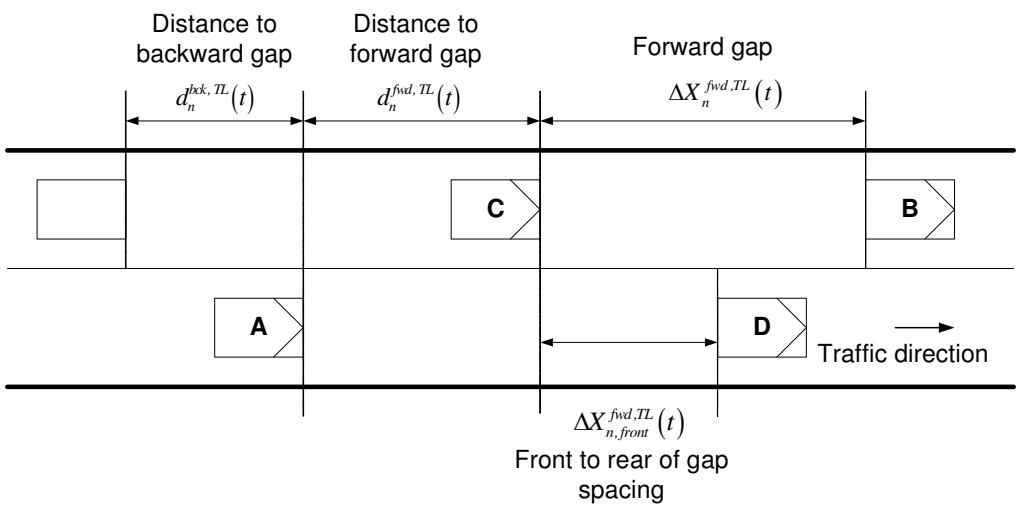

Figure 5 The effective gap and distances to the forward and backward gaps

The gap rate of change is defined as the speed of the vehicle at the rear of the gap (C) less the speed of the vehicle that determines the length of the effective gap, (either B or D, D as drawn). A positive rate of change implies that the gap is getting smaller, whereas a negative value implies that it is getting larger. The estimated coefficient of this variable is negative, suggesting that drivers try to anticipate the evolution of gaps when choosing a target gap. The front vehicle dummy variable captures the effect of the presence of the front vehicle on the target gap choice. It is equal to 1 when

the front vehicle is the constraining vehicle in defining the effective gap length (in Figure 5, this would be the case for the forward gap but not for the adjacent gap), and 0 otherwise. Although it is significant compared to other parameters, as expected, the estimated coefficient for this variable is negative, which indicates that drivers prefer to avoid having to consider the front vehicle as well when negotiating the lane change. The estimated coefficients of υ_n, for the forward and adjacent gaps are both positive. This result is consistent with the positive correlation of υ_n with timid drivers who are more likely to choose the adjacent and backward gap over the forward gap relative to more aggressive drivers. Therefore, these parameters are retained in the model although they are not statistically significant.

Gap choice probabilities as a function of the gap lengths, rate of change of the adjacent gap and distances to gaps are shown in Figure 6. Unless varied, the lengths of all gaps are 5 meters, the rates of change are zeros and the distances to the forward and backward gap are equal.

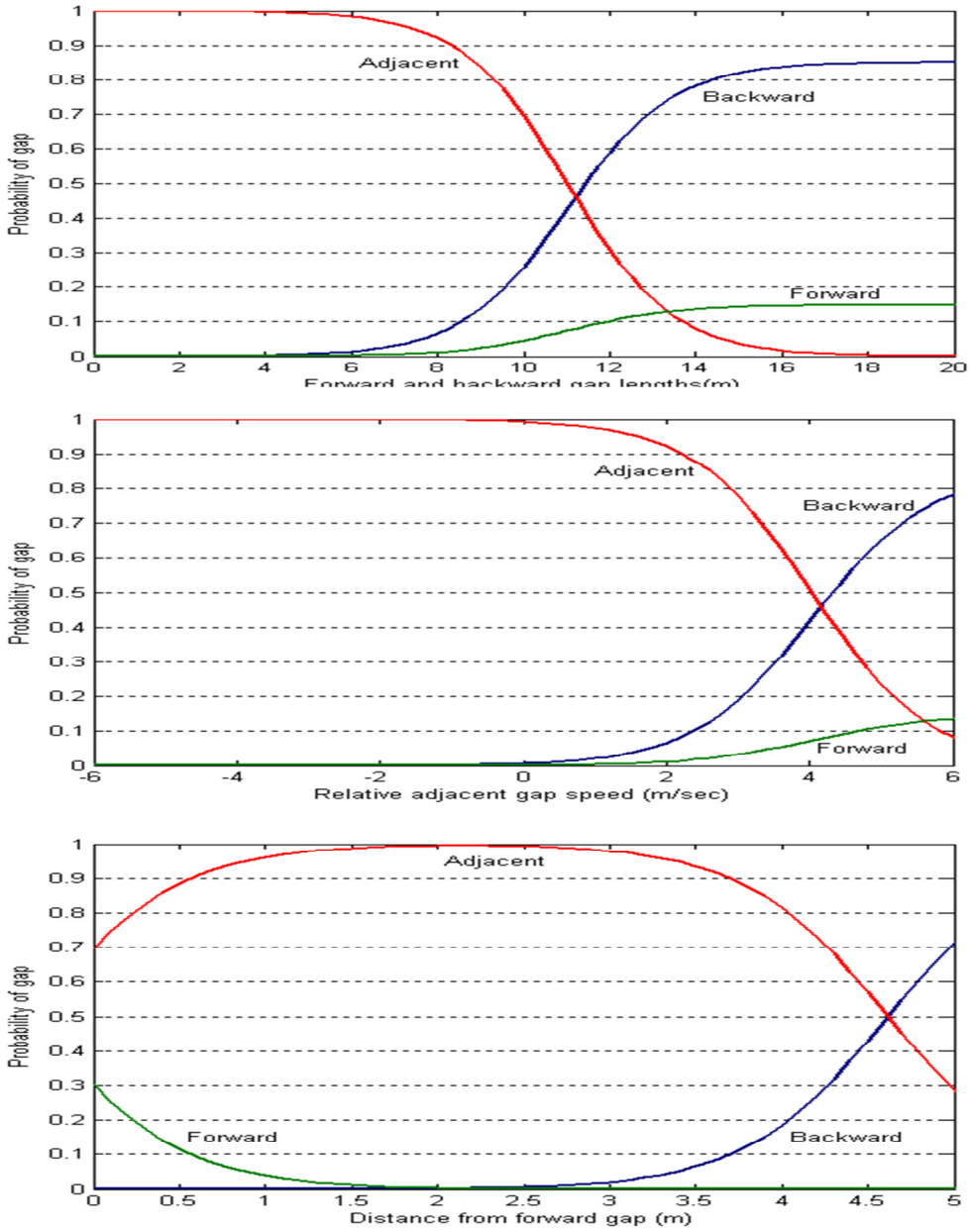

Figure 6 Gap choice probabilities as a function of gap lengths, rate of change of the adjacent gap and distance from the forward gap

4.4 Acceleration models

The acceleration model captures the behavior in a number of different situations, which depend on the driver's choices of target lane, lane changing and target gaps. These situations are for drivers that choose to stay

in their lane, drivers that execute a lane change, and drivers that plan to change lanes using a target gap. In order to capture the effect of the subject's leader, two driving regimes, car-following and unconstrained, are defined within each one of these situations. In the car following regime the subject vehicle is close to its leader and reacts to it. In the unconstrained regime does not react to the leader but accelerates to facilitate the short-term plan. The driving regime is determined by a threshold on the time headway between the subject and the leader:

$$a_n^k(t) = \begin{cases} a_n^{cf}(t) & \text{if } h_n(t-\tau_n) \le h_n^* \\ a_n^{k,uc}(t) & \text{otherwise} \end{cases} \tag{19}$$

$a_n^{cf}(t)$ and $a_n^{k,uc}(t)$ are the car-following and unconstrained acceleration that applies in situation *k*, respectively. τ_n is the reaction time of driver *n*. $h_n(t-\tau_n)$ and h_n^* are the time headway at time $t-\tau_n$ and the headway threshold for driver *n*, respectively.

The stimulus-sensitivity framework proposed within the GM model (Gazis et al. 1961) is adapted for all these acceleration models. Thus, the acceleration driver *n* applies in each cases *r* is assumed to be a response to the relevant stimulus from the environment:

$$a_n^r(t) = sensitivity_n^r(t) \times stimulus_n^r(t-\tau_n) + \varepsilon_n^r(t) \tag{20}$$

$\varepsilon_n^r(t)$ are random error terms.

<u>Car following model</u>

The functional forms of the car following stimulus and sensitivity functions follow Ahmed (1999) who extended the non-linear GM model (Gazis et al. 1961). The model also differentiates between acceleration and deceleration situations based on the sign of the relative leader speed. The stimulus and sensitivity functions are given by:

$$f^g\left[\Delta V_n(t-\tau_n)\right] = \left|\Delta V_n(t-\tau_n)\right|^{\lambda^g} \tag{21}$$

$$s^g\left[X_n^g(t)\right]=\alpha^g V_n(t)^{\beta^g}\,\Delta X_n(t)^{\gamma^g}\,k_n(t)^{\rho^g} \tag{22}$$

$g=acc,\ dec$. $\Delta V_n(t)$, $V_n(t)$ and $\Delta X_n(t)$ are the relative leader speed, subject speed and the spacing between the subject and its leader, respectively. $k_n(t)$ is the traffic density ahead of the subject. λ^g, α^g, β^g, γ^g and ρ^g are parameters.

The car following model is also applied to capture the behavior of drivers during the time a lane change is executed. It is assumed that in these situations vehicles follow the leader in the lane they are changing to. Thus, the lane changing car following model is given by:

$$a_n^{lc,cf,g}(t)=\alpha^g V_n(t)^{\beta^g}\,\Delta X_n^{lead,TL}(t)^{\gamma^g}\,k_n(t)^{\rho^g}\left|\Delta V_n^{lead,TL}(t-\tau_n)\right|^{\lambda^g}+\varepsilon_n^{cf,g}(t) \tag{23}$$

$\Delta X_n^{lead,TL}(t)$ and $\Delta V_n^{lead,TL}(t)$ are the spacing and relative speed with respect to the leader in the target lane.

Estimation results for the car following model are summarized in Table 4. The effects of different variables on the mean car following acceleration and deceleration are shown in Figure 7 and Figure 8, respectively. In these figures the following default values are assumed: the subject speed is 15 m/sec., space headway is 25 meters, density is 30 veh/km/lane and the absolute value of the relative leader speed is 3 m/sec.

Car following acceleration and deceleration both increase (in absolute value) with the relative leader speed stimulus. The magnitude of sensitivity to a negative stimulus (i.e. when the leader is slower) is much larger than the sensitivity to a positive one. This is expected since negative stimuli are associated with crash risk, whereas positive stimuli only suggest a possible speed advantage to the driver. The magnitudes of both the acceleration and deceleration drivers apply decreases when the space headways are larger. For deceleration car following this is expected since the underlying crash

risk is lower with larger spacing. In acceleration car following, it may be related to a reduced perception of the leader as a stimulus the driver needs to react to. The subject's speed affects the acceleration model, but not the deceleration model. The acceleration drivers apply increases with their speed. Finally, both accelerations and decelerations increase with traffic density.

Table 4 Estimation results for the car following model

	Acceleration		Deceleration	
Variable	Parameter value	t-statistic	Parameter value	t-statistic
Relative speed, m/sec.	0.520	7.97	0.834	12.68
Sensitivity constant	0.0355	1.21	-0.860	-3.92
Space headway, m.	-0.166	-1.68	-0.565	-9.51
Speed, m/sec.	0.291	5.64	-	-
Density, veh/km/lane	0.550	2.50	0.143	2.04
$\ln\left(\sigma_{cf,\,acc}\right)$	0.126	12.05	0.156	14.87

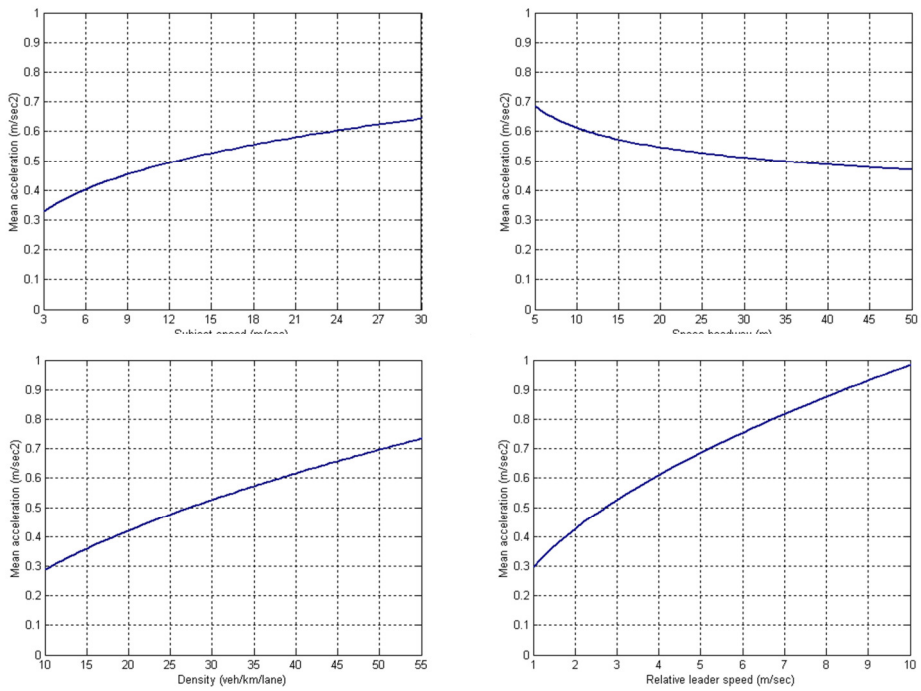

Figure 7 Effects of different variables on the mean car following acceleration

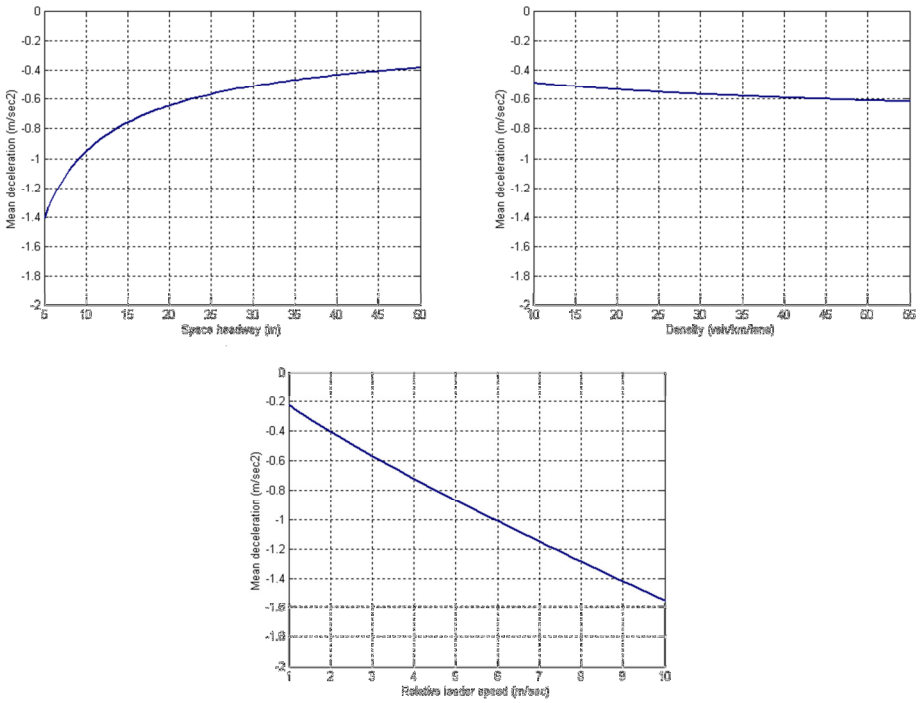

Figure 8 Effects of different variables on the mean car following deceleration

The free-flow model

In the free-flow regime, the stimulus is the difference between the desired speed and the actual speed. A constant sensitivity term is assumed, and so the acceleration is given by:

$$a_n^{ff}(t) = \lambda^{ff}\left[V_n^{DS}(t-\tau_n) - V_n(t-\tau_n)\right] + \varepsilon_n^{ff}(t) \tag{24}$$

$a_n^{ff}(t)$ is the free-flow acceleration driver *n* applies at time *t*. λ^{ff} is a constant sensitivity term. $V_n^{DS}(t-\tau_n)$ is the subject's desired speed, which is modeled as a linear function of explanatory variables. $\varepsilon_n^{ff}(t)$ is a random term.

Estimation results for the free flow model are summarized in Table 5. The heavy vehicle dummy variable captures the difference in desired speeds between heavy vehicles and other vehicles. The result indicates that the desired speed of heavy vehicles is lower by 1.458 m/sec (5.2 km/h). The effect of υ_n on the desired speed is negative. This is consistent with the positive correlation between this variable and the driver's timidity.

Table 5 Estimation results for the free flow model

Free-flow acceleration		
Sensitivity constant	0.0881	11.20
$\ln(\sigma_{ff})$	0.169	10.36
Desired speed		
Constant	17.636	61.26
heavy vehicle dummy	-1.458	-1.12
α^{DS}	-0.105	-0.40

Target gap acceleration model

Drivers that have selected a target gap to execute a lane change and are not constrained by their leaders choose accelerations that facilitate the short-

term plan. The model assumes that drivers have a desired position with respect to the target gap that would allow completing the lane change. This desired position is expressed as a fraction of the total length of the gap. The stimulus that drivers react to is the difference between the current position of the vehicle and this desired position. For the forward and backward gap accelerations, non-linear stimulus functions are used:

$$f^{TG}\left[D_n^{TG,TL}\left(t-\tau_n\right)\right]=\left(D_n^{TG,TL}\left(t-\tau_n\right)\right)^{\gamma^{TG}} \quad TG=fwd,bck \tag{25}$$

$D_n^{TG,TL}\left(t-\tau_n\right)$ is the distance to the desired position. γ^{TG} is a parameter.

The target gap sensitivity function depends on the subject speed and the target lane leader relative speed. The functional form used for the forward gap and backward gap accelerations sensitivities are, respectively:

$$s^{fwd}\left[X_n^{fwd}\left(t\right)\right]=\alpha^{fwd}V_n\left(t\right)^{\beta^{fwd}}\ exp\left[\lambda_+^{fwd}\Delta V_n^{lead,TL}\left(t\right)_+\right]exp\left[\lambda_-^{fwd}\Delta V_n^{lead,TL}\left(t\right)_-\right] \tag{26}$$

$$s^{bck}\left[X_n^{bck,TL}\left(t\right)\right]=\alpha^{bck}V_n\left(t\right)^{\beta^{bck}}\ exp\left(\lambda_+^{bck}\Delta V_n^{lag,TL}\left(t\right)_+\right)exp\left(\lambda_-^{bck}\Delta V_n^{lag,TL}\left(t\right)_-\right) \tag{27}$$

α^{fwd}, β^{fwd}, λ_+^{fwd}, λ_-^{fwd} and α^{bck}, β^{bck}, λ_+^{bck}, λ_-^{bck}, γ^{bck} are parameters.

$\Delta V_n^{lead,TL}\left(t\right)_+$ and $\Delta V_n^{lead,TL}\left(t\right)_-$ are the positive and negative relative target lane leader speeds, respectively. $\Delta V_n^{lag,TL}\left(t\right)_+$ and $\Delta V_n^{lag,TL}\left(t\right)_-$ are the positive and negative relative target lane lag speeds, respectively. These are defined: $\Delta V_n^{g,TL}\left(t\right)_+=max\left(0,\Delta V_n^{g,TL}\left(t\right)\right)$ and $\Delta V_n^{g,TL}\left(t\right)_-=min\left(0,\Delta V_n^{g,TL}\left(t\right)\right)$, $g\in\left\{lead,lag\right\}$.

This formulation allows different sensitivities of the acceleration to the relative speed of the leader in lane when the leader is faster or slower than the subject. The exponential form guarantees continuity of the acceleration when the relative target leader speed approaches zero.

For the adjacent gap acceleration, a constant sensitivity term α^{adj}, and a linear stimulus function are used:

$$f^{adj}\left[X_n^{adj,TL}\left(t-\tau_n\right)\right]=\beta^{DP}\Delta X_n^{adj,TL}\left(t-\tau_n\right)-\left(\Delta X_n^{lag,TL}\left(t-\tau_n\right)+l_n\right) \quad (28)$$

$\Delta X_n^{adj,TL}\left(t-\tau_n\right)$ is the clear adjacent gap spacing. $\Delta X_n^{lag,TL}\left(t-\tau_n\right)$ is the target lane lag space headway. l_n is the length of the subject vehicle. β^{DP} is the desired position relative to the gap expressed as a fraction of the total length of the gap (measured from the front of the gap lag vehicle).

Estimation results for these models are summarized in Table 6. The mean forward gap acceleration is positive and increases with the distance to the desired position. The forward acceleration also increases with the target lane leader relative speed. This is expected since drivers that target the forward gap must overtake the target lane leader to be able to merge into the forward gap, and so need to accelerate more aggressively when the target lane leader is faster (positive relative speed). The backward gap acceleration is negatively correlated with the distance to the desired position. This may be because drivers prefer to maintain their speed relative to the lead and lag vehicles to facilitate gap acceptance. The backward acceleration also decreases with the relative target lane leader speed. Drivers that target the backward gap must let the lag vehicle overtake them and so need to decelerate more aggressively when the target lane lag is slower (negative relative speed). The adjacent gap acceleration is positively correlated with the mis-positioning of the vehicle, i.e., the difference between the current position of the vehicle and the desired position.

Figure 9, Figure 10 and Figure 11 show the accelerations predicted by the forward, backward and adjacent gap acceleration models, respectively, for different relative leader speeds and distances to the desired position. Accelerations predicted by the free-flow model, which would have been used if the short-term plan was not modeled, are also shown. Target gap accelerations indicate more aggressive behavior compared to free flow accelerations. They are higher (in absolute value) for forward and backward

target gaps, and exhibit and strongly depend on the vehicle position for the adjacent gap acceleration.

Table 6 Estimation results for the target gap acceleration model

Variable	Parameter value	t-statistic
Forward gap acceleration		
Constant	0.385	1.39
Distance to desired position, m.	0.323	2.03
$\exp\left(\Delta V_n^{lead,TL}(t)_+\right)$, m/sec.	0.0678	1.13
$\exp\left(\Delta V_n^{lead,TL}(t)_-\right)$, m/sec.	0.217	-2.52
$\ln\left(\sigma_{fwd}\right)$	-0.540	-0.72
Backward gap acceleration		
Constant	-0.596	-1.56
Distance to desired position, m.	-0.219	-3.34
$\exp\left(\Delta V_n^{lag,TL}(t)_+\right)$, m/sec.	-0.0832	-1.15
$\exp\left(\Delta V_n^{lag,TL}(t)_-\right)$, m/sec.	-0.170	1.44
$\ln\left(\sigma_{bck}\right)$	0.391	1.86
Adjacent gap acceleration		
Constant	0.131	2.29
$\ln\left(\sigma_{adj}\right)$	-1.202	-2.50
Desired relative position		
Constant	0.604	5.59

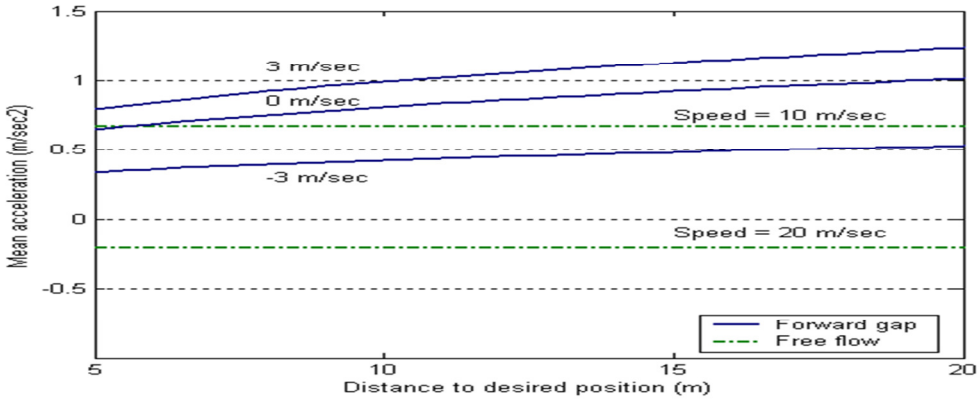

Figure 9 Predicted forward gap and free-flow accelerations

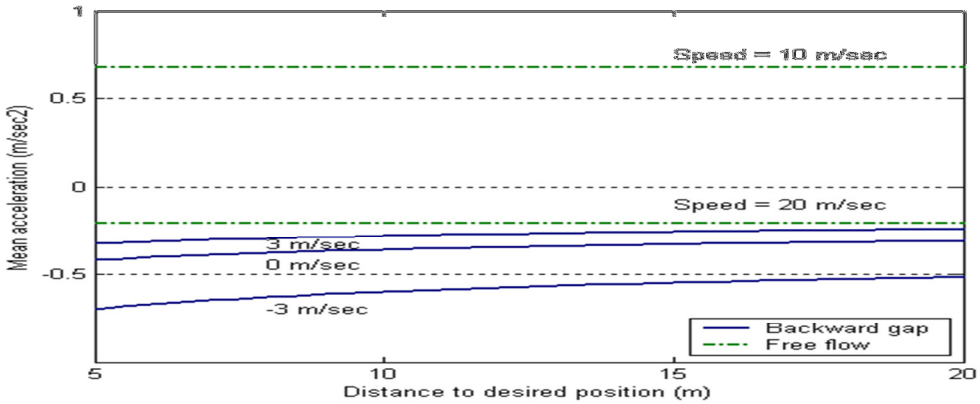

Figure 10 Predicted backward gap and the free-flow accelerations

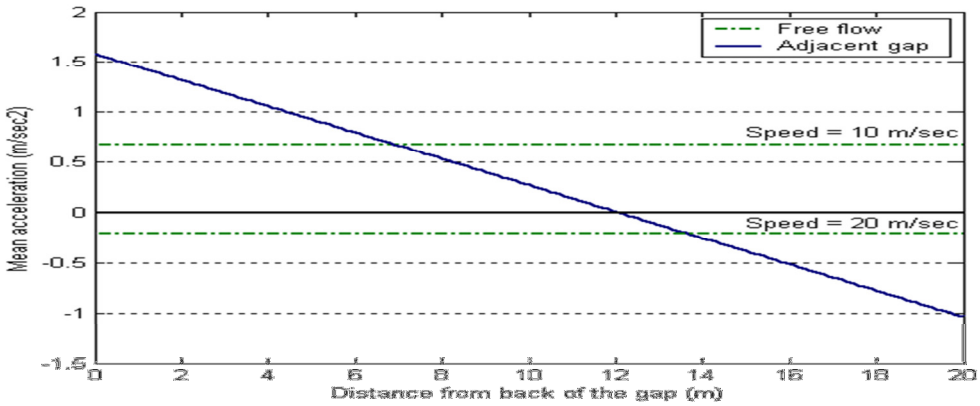

Figure 11 Predicted adjacent gap acceleration and free-flow accelerations

Distribution parameters

All components of the acceleration model are conditional on two driver characteristics: the reaction time and the headway threshold. Both are modeled as random variables to capture heterogeneity among drivers. Estimation results for these distributions are presented in Table 7. The median, mean and standard deviation of the reaction time distribution are 0.85, 1.10 and 1.00 seconds, respectively. These values are consistent with those reported in the literature.

Table 7 Estimation results for the reaction time and headway threshold distributions

Variable	Parameter value	t-statistic
Reaction time distribution		
Constant	-0.160	-3.08
$\ln(\sigma_\tau)$	-0.294	-1.20
Headway threshold distribution		
Constant	2.579	45.85
$\ln(\sigma_h)$	-0.799	-7.87

5 Conclusion

This paper presents the methodology and results of estimation of an integrated driving behavior model that explains lane changing and acceleration decisions jointly and so, captures inter-dependencies between these behaviors and represents drivers' planning capabilities. The parameters of all components of the model are estimated simultaneously using the maximum likelihood method. The values of the estimated parameters are tied to the data resolution of 1 second, and so the values presented here can only be used as they are in traffic simulators that implement this time step. However, the models can be implemented in traffic simulators that utilize different time steps, with adjustments only to

the model constants and variance parameters while keeping other parameters at their estimated values.

The model presented in this paper attempts to formulate a model that integrates various driving decisions. The target gap choice and target gap acceleration models are formulated in this paper for the first time. The underlying assumptions and specifications of the different components of these models need to be further studied, with different datasets and under different traffic conditions. For example, the specification presented assumes that the reaction time and time headway threshold distributions are identical in all acceleration behaviors. However, accelerations applied to facilitate lane changing may exhibit shorter reaction times (or in theory even anticipate conditions before they occur) because these are planned behaviors. Similarly, the target gap acceleration models presented here assume that drivers apply car following behaviors in the constrained regime. However, drivers may consider their lane changing goal even when they are constrained by their leader. The impact of these assumptions and the possibility of relaxing them should be further investigated. Finally, the model does not explicitly capture state-dependence among the short-term goals and plans a given driver chooses over time, which is computationally expensive to model. Methods to overcome this computational problem need to be developed.

The results presented in this paper are based on data collected in the 1980s in a single section of road. Until recently this was the only widely available dataset that could be used for this purpose. Recent data collection efforts that have been made within the NGSIM project (e.g. Cambridge systematic 2005) yielded new trajectory datasets that may also be useful in order to test the stability of the parameters of the various models over time and in different locations and to study the impact of the data time resolution. The question of the degree of transferability of the model

parameters to other locations has important implications on the practical application of models such as those presented in this paper.

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