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MICRO-SIMULATION OF URBAN NETWORKS

Moshe Ben-Akiva, Massachusetts Institute of Technology, Cambridge, MA
Margaret Cortes, Massachusetts Institute of Technology, Cambridge, MA
Angus Davol, Massachusetts Institute of Technology, Cambridge, MA
Haris Koutsopoulos, Volpe National Transportation Systems Center, Cambridge, MA
Tomer Toledo, Massachusetts Institute of Technology, Cambridge, MA

Published in in World Transport Research, Park C.H., Cho R., Oh J., Hayashi Y. and Viegas J. eds., Pergamon Press, 2003

INTRODUCTION

Urban and freeway networks differ significantly both in the physical nature of the network and the behavior of drivers. Compared to freeway networks, urban networks are characterized by shorter links and more significant effects from intersections and their control (both signalized and unsignalized) on delays and levels of service. The presence of public transit (especially when bus stops are frequent), bicyclists, pedestrians, and on-street parking further complicate the modeling of urban networks. These differences are especially important in the context of microscopic traffic simulation. Hence, models developed for freeway networks may not be transferable to urban settings.

In order to model urban networks accurately, a simulation model must incorporate the elements of these networks and also have realistic models for drivers' response to these elements. The simulation package MITSIMLab has been previously applied and validated primarily on freeway networks. New applications have prompted the further development of its urban modeling capabilities. In this paper we discuss some of the important issues in modeling urban networks and enhancements that have been implemented in MITSIMLab and validated in a real-world application.

The rest of the paper is organized as follows: An overview of the MITSIMLab simulation model is presented first. Next, we describe recent enhancements to improve the urban capabilities of the model, namely modeling of unsignalized intersections and roundabouts and modeling drivers' path awareness, as well as enhancements to the modeling of traffic signal control and public transportation. Finally these enhancements are demonstrated with a case study in Stockholm, Sweden.

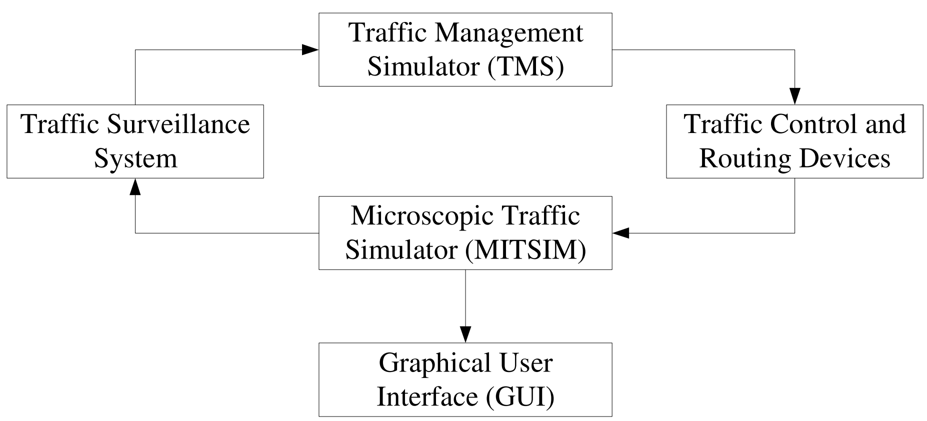
OVERVIEW OF MITSIMLAB

MITSIMLab (Yang et al., 2000) is a simulation-based laboratory that was developed for evaluating alternative traffic management system designs at the operational level and assisting in subsequent refinement. Examples of systems that can be evaluated with MITSIMLab include advanced traffic management systems (ATMS) and route guidance systems.

As a synthesis of a number of different models, MITSIMLab is able to represent a wide range of traffic management system designs, model the response of drivers to real-time traffic information and controls, and incorporate the dynamic interaction between the traffic management system and the drivers on the network.

The various components of MITSIMLab are organized in three modules: microscopic traffic simulator (MITSIM), traffic management simulator (TMS) and graphical user interface (GUI).

Figure 1. Elements of MITSIMLab and their Interactions



The interactions among the various MITSIMLab modules are shown in Figure 1. A microscopic simulation approach, in which movements of individual vehicles are represented, is adopted for modeling traffic flow in the traffic flow simulator (MITSIM). This level of detail is necessary for an evaluation at the operational level. The traffic management simulator (TMS) represents the candidate traffic control and routing logic under evaluation. The control and routing strategies generated by the traffic management module determine the status of the traffic control and route guidance devices. Drivers respond to the various traffic controls and guidance while interacting with each other. Output from the simulation can be obtained both in the form of raw data and via the graphical user interface (GUI), which is used for both debugging purposes and demonstration of traffic impacts through vehicle animation.

Traffic Flow Simulator (MITSIM)

The role of MITSIM is to represent the “world”. The traffic and network elements are represented in detail in order to capture the sensitivity of traffic flows to the control and routing strategies. The main elements of MITSIM are the following:

Network Components: The road network along with the traffic controls and surveillance devices are represented at the microscopic level. The road network consists of nodes, links, segments (portions of links with uniform geometric characteristics), and lanes.

Travel Demand and Route Choice: The traffic simulator accepts as input time-dependent origin to destination trip tables. These OD tables represent either expected conditions or are defined as part of a scenario for evaluation. A probabilistic route choice model is used to capture drivers’ route choice decisions. Route choices are based on real-time traffic conditions for drivers with access to information (via in-vehicle units or variables message signs) and on historic travel times for other drivers.

Driving Behavior: The origin/destination flows are translated into individual vehicles wishing to enter the network at a specific time. Behavior parameters (such as desired speed, aggressiveness, etc.) and vehicle characteristics are assigned to each vehicle/driver combination. MITSIM moves vehicles according to car-following and lane-changing models. The car-following model captures the response of a driver to conditions ahead as a function of relative speed, headway, and other traffic measures. The lane-changing model distinguishes between mandatory and discretionary lane changes. Merging, drivers’ responses to traffic signals, speed limits, incidents, and tollbooths are also captured. For a detailed description of driving behavior models, see Ahmed (1999).

Traffic Management Simulator (TMS)

The traffic management simulator mimics the traffic control system in the network under consideration. A wide range of traffic control and route guidance systems can be simulated, such as ramp control, freeway mainline control (e.g., lane control signs, variable speed limit signs, portal signals at tunnel entrances), intersection control, variable message signs and in-vehicle route guidance.

TMS has a generic structure that can represent different designs of such systems with logic at varying levels of sophistication, from isolated pre-timed signals to real-time predictive systems. Control strategies and routing information are generated using either a *reactive* or *proactive* approach. The reactive approach consists of pre-determined control laws that depend only on the current network state. In the proactive approach, the system predicts future traffic conditions and optimizes traffic control and routing strategies based on this prediction. In this case, the generation of control and routing strategies is an iterative process. Given a proposed strategy, traffic conditions on the network are predicted and the performance of the candidate strategy is evaluated. If the strategy is found to be satisfactory, the strategy is implemented; if additional strategies need to be tested, another generation-prediction iteration is performed.

MODELING URBAN NETWORKS

Several enhancements have been made to improve the urban network modeling capabilities of MITSIMLab. These enhancements can be organized in two categories: (1) behavioral models that better capture drivers' behavior in situations encountered in urban environments; and (2) functionality enhancements to better represent elements of the urban transportation system.

This section describes in detail enhancements to two of the behavioral models: drivers' behavior at intersections and drivers' path-awareness capabilities and to the representation of traffic signal control and public transit operations.

Modeling of Driver Behavior at Intersections

Intersections are among the most important features of urban networks. Interactions between vehicles in the intersection area cause delays to vehicles. These delays make up a significant proportion of the total trip time in urban networks. Moreover, this proportion increases with the level of congestion in the network, as growing numbers of vehicles in intersecting links compete over limited capacity at the intersection.

Drivers approaching an intersection are confronted with the decision of how to negotiate their intended maneuver in the intersection. To make this decision, drivers have to assess vehicle positions and speeds in other approaches to the intersection as well as their own desired speed within the intersection.

Existing Models
Traditionally, this behavior is modeled with gap acceptance models. These models are based on the notions of priorities and conflicting movements in the intersection. Movements in the intersection are ranked in terms of right of way, and conflicts between different movements are identified. Movements with lower priority are assumed to yield to higher priority movements, waiting for sufficient gaps in higher priority conflicting traffic. Gap acceptance models are used to determine their actions on the existing gaps. These models formulate a problem in which a driver is presented with a gap. The choice set is binary, i.e. the driver will either accept or reject the gap.

The decision is based on comparison of the existing gap with the critical gap, the unobservable minimum acceptable gap. Mathematically it can be written as

$$Y_n(i) = \begin{cases} 1 & \text{if } g_n(i) \geq g_n^{cr}(i) \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

Where $Y_n(i)$ is an indicator variable for the decision driver *n* takes on gap *i,* and $g_n(i)$ and $g_n^{cr}(i)$ are the gap and critical gap lengths, respectively. A value of 1 for $Y_n(i)$ indicates that the gap is accepted while a value of 0 indicates that the gap is rejected.

Critical gaps are assumed to be random variables drown from some distribution (e.g., Cohen et al., 1955; Miller, 1972). Mahmassani and Sheaf (1981) allowed the mean of the critical gap distribution to be a function of explanatory variables. This enabled introducing impatience functions, in which critical gaps are decreasing functions of delay, such as waiting times at the stop line or the number of rejected gaps.

Another class of models used for intersection modeling is that of real-estate models. One such model was implemented in the simulation package CORSIM (FHWA, 1998). The model does not explicitly model drivers' behavior, but instead uses a cellular-automata-like representation of the intersection to determine the ability of a vehicle to perform its movement. The intersection area is divided into blocks of 15-20 square feet in which a vehicle may reside. Each vehicle movement within the intersection is directly related to certain real-estate blocks. A vehicle's path is defined by a sequence of blocks that it occupies as it moves through the intersection and by the duration that each block is occupied. A vehicle within or approaching an intersection influences blocks along its path, both upstream and downstream of its position. The extent of influence varies with the speed and acceleration of the vehicle and is determined by simple rules. These influences define the status of each block at every time interval. A block may be free, occupied, or under influence of a vehicle prohibiting other vehicles to occupy it.

The advantage of this approach is that it does not require enumerating all possible combinations of vehicle paths within the intersection that can create conflicts a priori. Instead, only the status of the real-estate blocks needs to be checked in order to identify conflicts. The shortcoming of this approach is in the use of simplistic occupancy rules and, even more seriously, the dependency of the outcome on the real-estate blocks representation of the intersection. It is almost impossible to formulate a general logic for defining blocks that would be able to handle intersections with non-standard geometry. This problem caused CORSIM to drop this approach for modeling intersections, and prevents its implementation in other models.

A Proactive Anticipatory Gap Acceptance Model

The binary formulation of intersection gap acceptance decisions may be reasonably suitable for drivers standing at the stop line. However, it is far less suited for moving vehicles or crawling traffic. The major deficiency is that drivers are assumed to be passive. In these models, their choice set is restricted to a simple accept/reject decision. In reality, however, drivers play a more active role and make more complicated choices. For example:

1. Drivers do not only make accept/reject decisions but also adjust their speeds and acceleration rates to enable themselves to accept gaps. Modeling this decision is essential for models used in a microscopic simulation context where the basic variables are speed and acceleration.
2. In the gap evaluation process, drivers do not make their decisions based on actual gaps, because these are unknown at the time the decision is made. Instead, they estimate available gaps by predicting the behavior of vehicles in conflicting movements.
3. Drivers' acceleration decisions are constrained by an upper bound on desired maneuvering speed. This speed may be a function of the type of maneuver (e.g., crossing, right turn, left turn) and the geometry of the intersection.

A model that explicitly addresses the above considerations has been implemented in MITSIMLab. In the first step of the model, the vehicle is tagged as approaching an intersection. This is done at a distance from the intersection related to the stopping distance of the vehicle and the visibility of the intersection. The tagging distance is randomly distributed over the population of drivers. Once a vehicle is tagged, it starts adjusting its speed to the intersection. A maximum desired maneuver speed is defined for each vehicle. This speed is based on the specifics of the maneuver the vehicle is about to undergo in the intersection and a random component.

A tagged vehicle starts to consider gaps along with the other constraints (car following, traffic signals, etc.) when making acceleration decisions. This primarily concerns the first vehicle in the lane approaching an intersection, but is also applied in a restricted way to following vehicles to allow several vehicles to use the same gap should the gap size allow it. The consideration of gaps consists of three components:

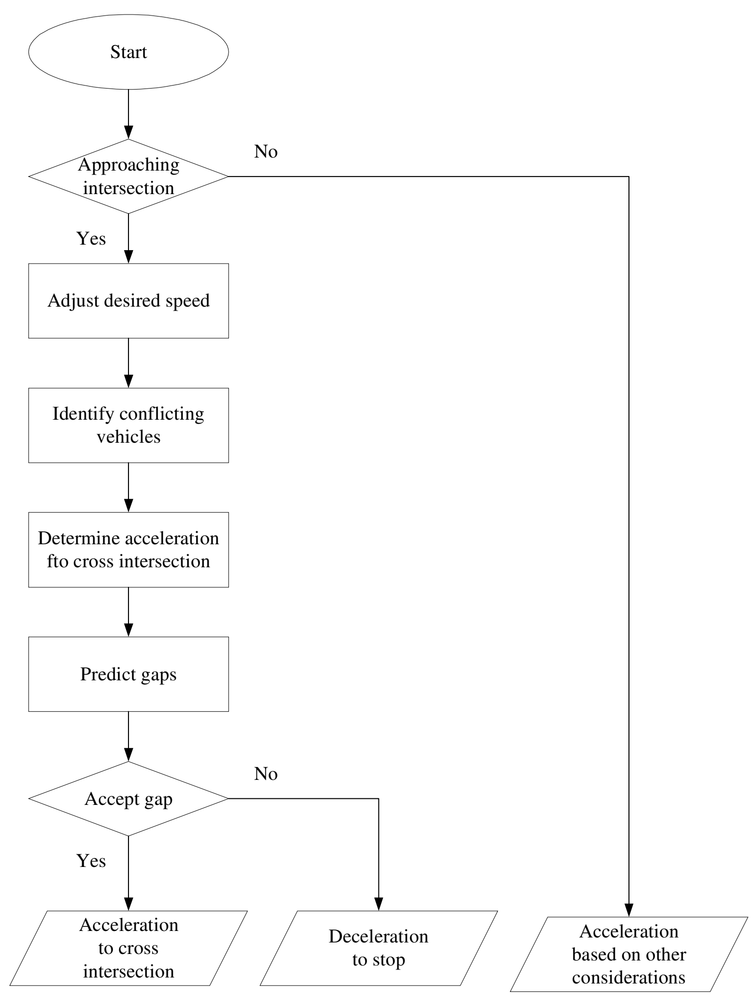
1. *Identifying conflicting vehicles*. The driver identifies conflicting vehicles that need to be yielded to. The implementation is based on the hierarchy of right-of-way in the intersection. The vehicle identifies the movements that have priority over its own, identifies the lanes that these movements would be using to approach the intersection, and identifies the first vehicle in each of these lanes.
2. *Predicting gaps*. Once the conflicting vehicles are identified, the driver predicts the available gaps. The prediction of gaps is based on the arrival times of the vehicle and the conflicting vehicles at the conflict zone. The vehicle's own arrival time prediction is conditional on the acceleration the driver would apply. Since the driver cannot know how other drivers will behave, the prediction of their arrival

times is based only on their current speeds. The gap to which the driver reacts is the minimum gap among all conflicting movements.

3. *Evaluating gaps and deciding acceleration*. Having predicted the available gap, the driver evaluates it, decides whether to accept it or not, and applies the appropriate acceleration. The gap acceptance decision is based on comparison of the existing gap with a critical gap. This critical gap is movement-specific, and randomly distributed between drivers.

For the default implementation in MITSIMLab, the critical gap is normally distributed around a mean specified as an input parameter. In the current implementation, the driver tries to maximize the probability that the available gap will be acceptable by adjusting speed and acceleration to create the largest possible gap. Other driving constraints set the limits of acceleration. If the gap is unacceptable, even under optimal arrival timing at the intersection, the driver will decelerate in order to be able to stop at the stop line. The logic is summarized in Figure 2.

Figure 2. Logic of the Enhanced Intersection Behavior Model



Modeling of Drivers' Path Awareness

Most lane changing models assume that two types of lane changes exist: discretionary and mandatory. A discretionary lane change is performed when the driver perceives that it will improve the driving experience (e.g., allow a faster speed or reduce interaction with other vehicles). A mandatory lane change is triggered when the driver must perform a lane change in order to follow its path (e.g., be on a lane that leads to an exit ramp from a freeway). The problem of path awareness arises in the context of mandatory lane changes. The question is when do drivers become aware of the path plan constraint that triggers a mandatory lane change.

The previous approach used in MITSIMLab was that at any time drivers are only aware of the next link on their path. Therefore only lane changes required in order for the vehicle to be able to continue to the next link are considered. This approach is common to many microscopic simulation models. It is mainly driven by the computational efficiency gained from the fact that a vehicle only needs to know the next link on its path rather then store information on the whole path.

The use of this approach may be reasonable for freeway networks, which have relatively long links. However, it is problematic in an urban environments, which are characterized by short links and paths that may require frequent turning movements. In such cases, the one-link awareness model generates excess weaving and merging maneuvers due to many late lane changes, therefore leading to an underestimation of capacity at these locations. Real-world drivers may consider their path plan well in advance and adjust their position to allow for a smoother continuation of the path.

A new awareness model that overcomes the limitations of the one-link awareness has been implemented in MITSIMLab. The model assumes that a driver is aware of the path-plan up to a certain distance downstream of the current position. The driver will react to any mandatory conditions that arise within this "look-ahead" distance and ignore any such considerations beyond that distance. The look-ahead distance is a characteristic of the driver, and may depend on factors such as familiarity with the path and spatial abilities. Look-ahead distances are assumed to be randomly distributed in the population of drivers. For the default implementation in MITSIMLab, the look-ahead distance is uniformly distributed between minimum and maximum values specified as input parameters.

The critical operation in the implementation of the look-ahead model is mapping the lane connectivity from the vehicle's position to the look-ahead distance downstream. This is required in order to determine whether a mandatory lane change is required and, if so, in which direction. This operation is a double pass process:

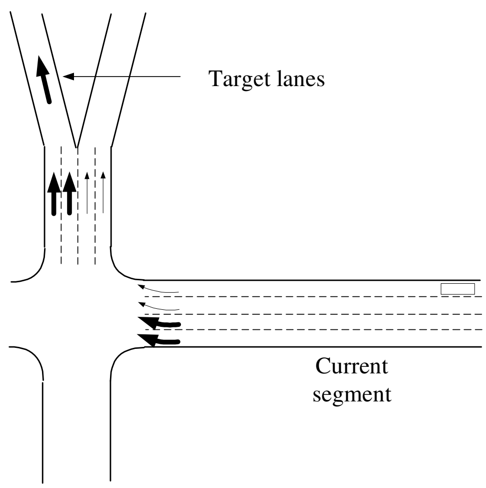
1. *Forward pass*. Starting at the position of the vehicle, the next segments on the path are accumulated up to the look-ahead distance. The lanes in the last segment

within the look-ahead distance are labeled as the target lanes. The driver will want to be in a lane that is connected to any of these lanes.

2. *Backward pass*. Starting from the target lanes and moving backwards on the path, a list of lanes connected to the target lanes is built. Next, the lanes on that list are used as target lanes, and the process is repeated for the upstream segment until the connected lanes in the current segment are identified.

The list of connected lanes in the current segment is used to determine whether a mandatory lane change is required. If the vehicle's current lane is connected, no lane change is needed. Otherwise a mandatory lane-change in the direction of the connected lanes is triggered. Figure 3 shows an example of a current segment, target lanes and connected lanes (shown in bold). The vehicle shown in this figure will immediately initiate a mandatory lane change to either of the two left lanes.

Figure 3. Mapping of lane connectivity



Functionality Enhancements

Additional components of MITSIMLab have been enhanced to further improve its urban network modeling capabilities. Major areas of enhancement are traffic signal control and modeling of public transit vehicles. Implementation of these elements allows the model to be used for applications such as analysis of transit operations and evaluation of signal priority strategies for transit vehicles.

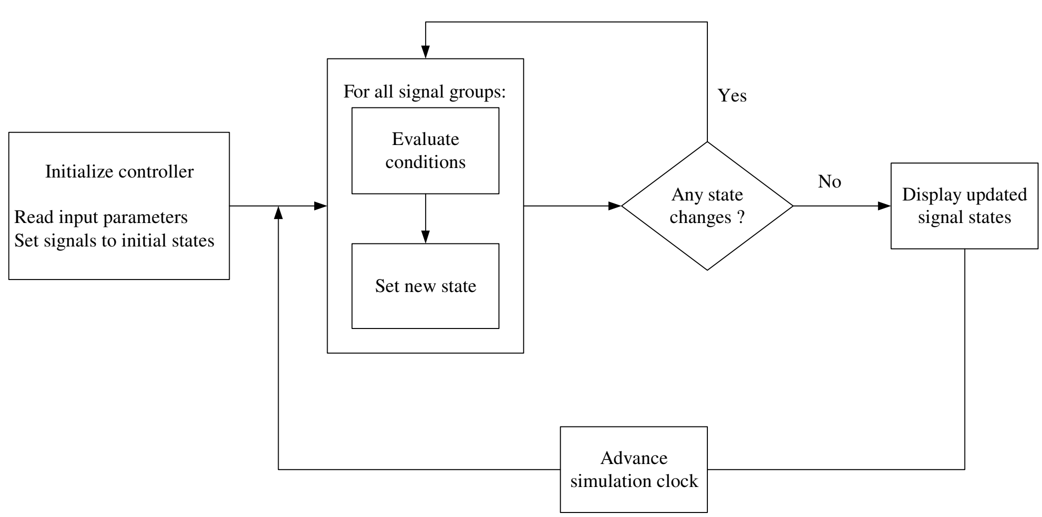
Traffic Signal Control

The traffic management simulator (TMS) module in MITSIMLab supports a wide range of signal control and route guidance systems, including freeway ramp and mainline controls, intersection traffic signal controls, variable message signs, and in-vehicle information. Traffic signal control at intersections originally supported two types of controllers: pre-timed and actuated. The logic for both of these controllers requires pre-specified phase plans and phase orders, which limits their usefulness in modeling more advanced control systems. For example, the modeling of dual-ring controllers was not possible, nor was the modeling of European controllers in which phasing is not necessarily explicit.

In previous applications of MITSIMLab, the basic controllers have been adequate. In some cases, such as for pre-timed or four-phase actuated controllers, the control logic can be simulated exactly. In other cases, the existing controllers have been used to approximate more advanced control logic.

New applications of MITSIMLab, however, will focus more directly on urban traffic control. A new generic controller has been implemented in MITSIMLab to address the limitations of the original controllers. It has been designed with a generic control logic that allows the simulation of a wide range of advanced control strategies. Instead of requiring each phase to be specified explicitly, the generic logic allows each movement to be controlled independently, thus allowing full flexibility in the control logic. This logic is specified by means of detailed conditions that must be met before a signal indication changes, such as time, detector states, other signal states, and other controller states. By specifying these conditions, any logic can be modeled. For example, specifying only time constraints simulates a pre-timed controller. Constraints with respect to detector states add vehicle actuation, while constraints as to other signal states allow the specification of complementary or conflicting movements. Conditions on other controller states allow coordinated and area-wide adaptive control, which is necessary for the simulation of advanced control systems. Figure 4 summarizes the logic of the generic controller. The implementation of this generic controller has improved the ability of MITSIMLab to serve as a laboratory for advanced traffic management systems.

Figure 4. Logic of the generic controller



Public Transit Operations

Enhancements to MITSIMLab to expand its transit modeling capabilities have also been implemented. The capabilities required for modeling transit operations fall into four categories: supply-side, demand-side, control, and output. The supply-side includes network components (such as bus stops, terminals, and bus lanes) and movement characteristics (such as vehicle properties, routing, and scheduling). With the added functionality MITSIMLab is able to simulate bus operations in great detail.

The transit implementation includes a schedule based supply model with detail input of vehicle schedules. Bus movements in the network are enhanced to recognize the presence of stops, stop configuration and the existence of bus lanes. In addition buses are able to trigger signal priority systems at intersections.

The demand-side components involve a simulation of passenger demand for transit. This is manifested primarily in the vehicle dwell times at stops. Several levels of demand representation have been identified, and are presented in increasing order of complexity. Depending on data availability the appropriate version can be used in application:

1. *No demand effect.* In this implementation, dwell times are pre-specified and therefore not subject to variation based on demand. This provides a basic representation of the effect of buses on traffic operations in terms of impedance to traffic flow while requiring minimal input data.
2. *Stop level demand.* At this level, demand is represented through arrival rates and percent alighting at stops. This representation allows for realistic modeling of dwell time at stops and taking into account dwell time interactions among stops.
3. *Network level demand.* The most advanced model uses network passenger data, such as a passenger origin-destination matrix, in place of stop-specific data. This allows the implementation of passenger demand models in order to capture the impact of operating policies.

In addition to the graphical display of the network, MITSIMLab provides output data at system, link, segment, sensor, and vehicle levels. The vehicle-level information (e.g., travel time and average speed) is valuable for transit applications. Transit specific information that allows the development of measures of effectiveness related to level of service and efficiency of operation is reported. For example, schedule adherence, service reliability, waiting times, crowding level may be calculated and used to evaluate the transit system.

With the transit simulation capabilities described above MITSIMLab is able to evaluate not only traditional transit systems but also advanced public transportation systems (APTS). Examples of such applications include:

- Evaluation of automated vehicle location (AVL) systems.
- Evaluation of real-time operations control strategies, such as holding.

- Evaluation of signal priority strategies both unconditional and conditional. Conditional strategies may be based on factors such as passenger load, schedule adherence, headway variability and various other conditions.

The modular structure of the implementation allows the simulation of both current and future APTS technologies.

CASE STUDY: STOCKHOLM, SWEDEN

The enhanced MITSIMLab model was applied to a network in Stockholm, Sweden, as part of an evaluation of MITSIMLab for use in that city. The network chosen for the evaluation was a mixed urban-freeway network in the Brunnsviken area, a map of which is shown in Figure 5. It includes part of the E4 motorway leading from downtown Stockholm north to the Arlanda International Airport and a parallel arterial. The urban part of the network includes geometrically complex intersections, both signalized and unsignalized, including a large traffic circle. The network also experiences extremely heavy peak period congestion, adding to the challenge of simulation. For an accurate simulation of the network, the urban modeling enhancements with respect to driving behavior implemented were found to be essential.

The most significant issues on the urban portions of the network were the short link lengths and complex intersections. Without the path-awareness model, unrealistic weaving behavior was occurring at some intersections. An example of such a situation is the Norrtull intersection, shown schematically in Figure 6. Vehicles traveling westbound on Sveavägen heading to the E4 motorway will turn right at Norrtull. A short distance (approximately 50 meters) later the road splits, with the right branch leading to the E4 northbound and the left branch to the E4 southbound. Figure 6a shows the behavior of a vehicle heading to the E4 southbound under the one-link awareness model. At Sveavägen, the right lane allows the vehicle to continue to the next link, and therefore the driver may choose to stay on that lane. Only after crossing Norrtull the driver is aware that a lane change is required in order to take the left branch at the divergence point. The driver will then need to perform lane changes in the short section before the divergence in order to follow the path. Because many drivers will have similar behaviors, the net effect will be a reduction in the capacity of the section. If demand is sufficiently large, a bottleneck will be created. Eliminating the one-link awareness restriction causes most drivers to consider their intended maneuver at the divergence before crossing Norrtull and to move to the left lane while on Sveavägen (as shown in Figure 6b). This is more representative of the actual behavior, where drivers' knowledge of the network and response to directional signs may contribute to a smoother flow of traffic.

Figure 5. Map of Stockholm Simulation Area

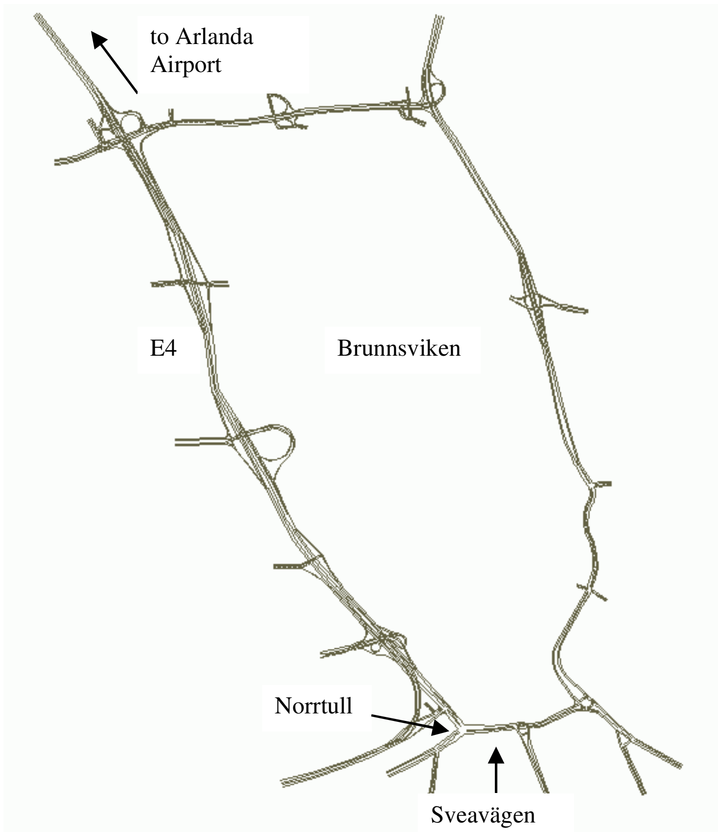
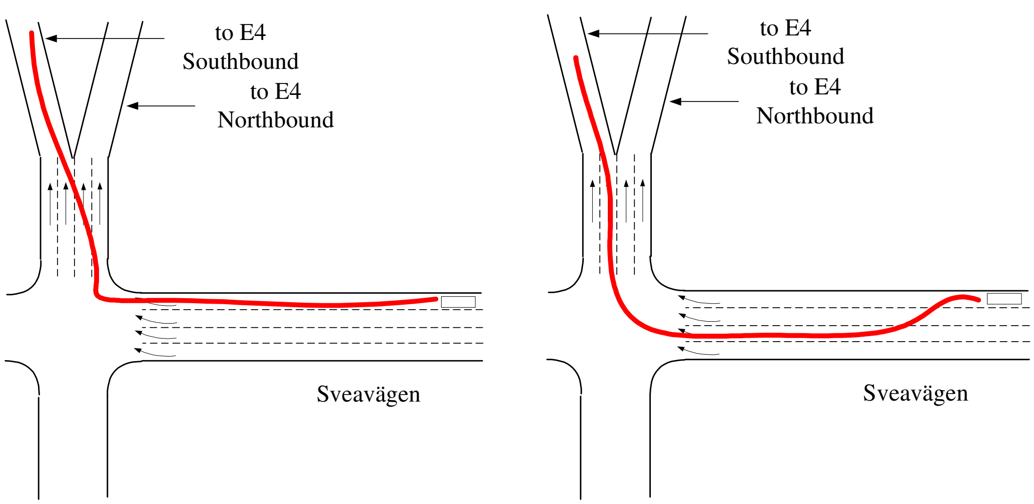


Figure 6. Schematic of the Norrtull intersection



(a) Without path awareness (b) With path awareness

To test the importance of the enhancements and their impact on the quality of the results, MITSIMLab was run on the network both with and without the enhancements implemented. All other model and input parameters were identical. Without the implementation of the path-awareness model In the MITSIMLab simulation model, the capacity of the approaches to Norrtull was significantly underestimated. Figure 7 compares the simulated and actual traffic counts for the two-hour simulation period on Sveavägen westbound, the intersection approach shown in Figure 6. Simulated counts are shown for three modeling conditions: (1) without path awareness, (2) with look-ahead distances distributed between 100 and 200 meters, and (3) with look-ahead distances distributed between 100 and 400 meters. Without the path-awareness model, excessive queues developed on the approach to Norrtull, leading to the reduced flows shown in Figure 7. For the path-awareness model implemented with look-ahead distances between 100 and 200 meters, queue lengths and flows are close to the actual conditions. Increasing the look-ahead range to 400 meters does not have a significant effect on the results, suggesting that the smaller range is adequate for urban streets.

Figure 7. Traffic Counts on Sveavägen Westbound

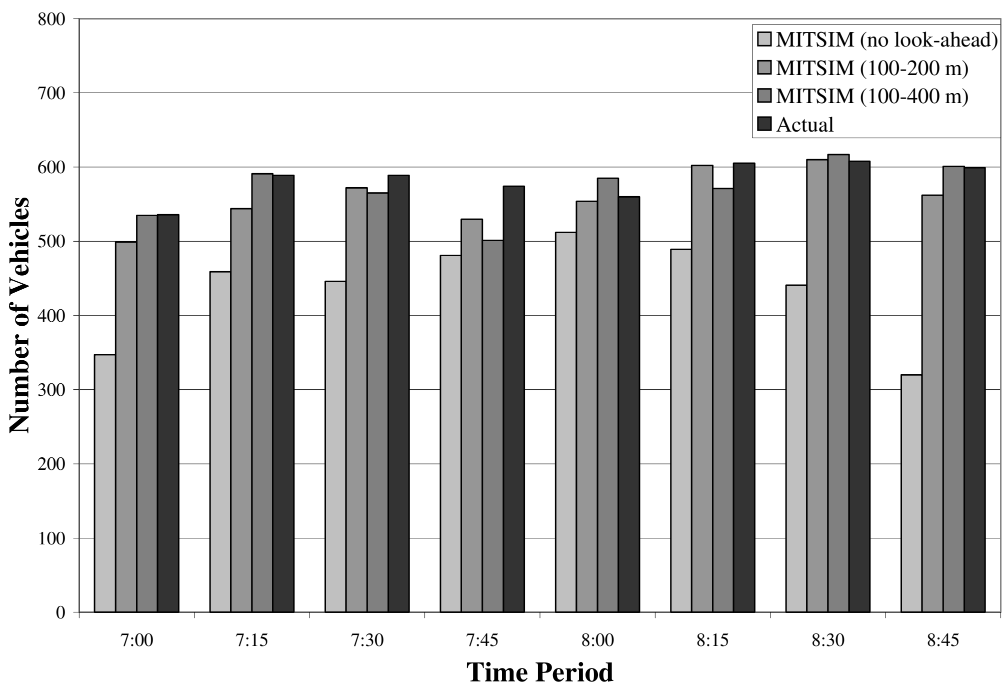
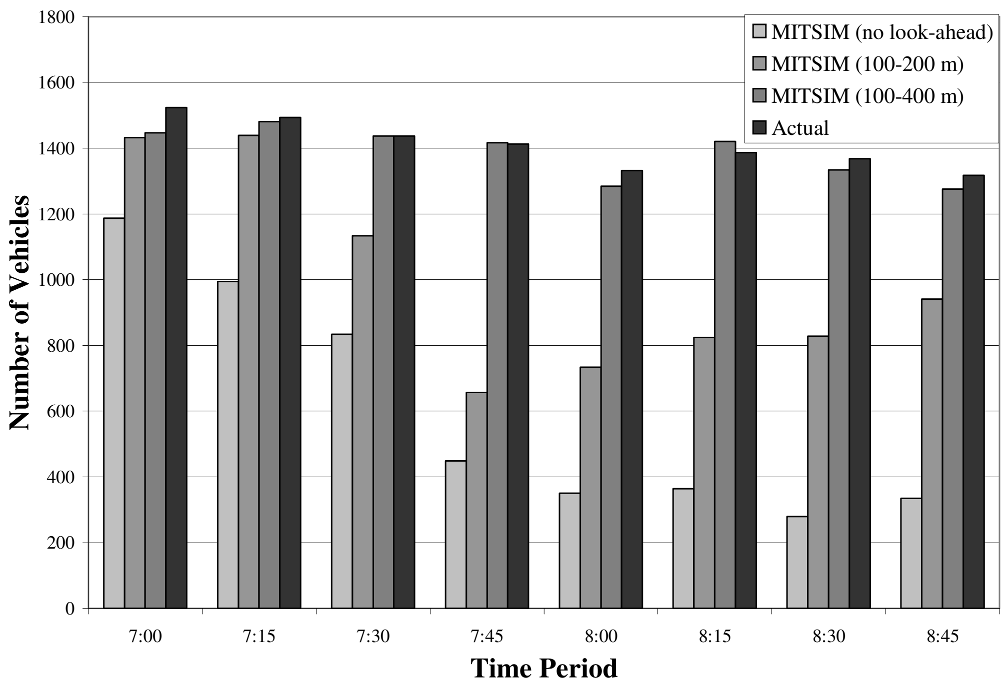


Figure 8 shows similar results for the northern approach to the Norrtull area, coming from the E4 motorway southbound. The count location is well north of Norrtull, but queues from Norrtull spill back onto the motorway without the implementation of the path-awareness model, leading to the reduced flows in the figure. With look-ahead distances distributed between 100 and 200 meters, flows on the motorway are increased but are still affected by the downstream bottleneck. When the upper bound on look-ahead distance is increased to 400 meters, the simulated flows closely match the actual measurements. A larger look-ahead distance is reasonable in this case because the approach is a freeway section, which should have directional signs well in advance of exits and intersections. This gives drivers advance warning of lane-changes that may become necessary.

Figure 8. Traffic Counts on E4 Motorway Southbound



The enhanced intersection driving behavior model was also found to be essential for the Brunnsviken network due to the presence of unsignalized intersections and roundabouts. With the intersection model implemented, MITSIMLab was able to represent the prioritization of movements that matched the actual site conditions.

CONCLUSION

Recent enhancements to the driving behavior models of MITSIMLab have improved the capability of MITSIMLab to model urban networks. As demonstrated by the application in Stockholm, these enhancements provide for a more accurate representation of driver behavior on urban streets. Functionality enhancements allow for simulation of a wide range of advanced traffic control concepts and APTS strategies and systems.

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