Driving Behaviors: Models and Challenges for Non-Lane based Mixed Traffic

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ABSTRACT

Most published microscopic driving behavior models, such as car following and lane changing, were developed for homogeneous and lane-based settings. In the emerging and developing world, traffic is characterized by a wide mix of vehicle types (e.g. motorized and non-motorized, two, three and four wheelers) that differ substantially in their dimensions, performance capabilities and driver behavior and by a lack of lane discipline. This paper presents a review of current driving behavior models in the context of mixed traffic, discusses their limitations and the data and modeling challenges that need to be met in order to extend and improve their fidelity. The models discussed include those for longitudinal and lateral movements and gap acceptance. The review points out some of the limitations of current models. A main limitation of current models is that they have not explicitly considered the wider range of situations that drivers in mixed traffic may face compared to drivers in homogeneous lane-based traffic, and the strategies that they may choose in order to tackle these situations. In longitudinal movement, for example, such strategies include not only strict following, but also staggered following, following between two vehicles and squeezing. Furthermore, due to limited availability of trajectory data in mixed traffic, most of the models are not estimated rigorously. The outline of modeling framework for integrated driver behavior was discussed finally.

KEYWORDS: Mixed Traffic; Longitudinal Movement Models; Lateral Movement Models; Trajectory Data; Model Calibration and Validation

1.0 INTRODUCTION

The rapid economic growth of developing and emerging countries has generated an increase in travel demand, overwhelming the limited transportation infrastructure. A useful indicator of that trend is the total number of motorized vehicles, which has increased from 1981 to 2012 from 5 million to 159 million in India and from 5.5 million to 221 million in China [1-2]. In both countries, the problem of increased motorization is compounded by an inadequate road infrastructure, unsafe vehicles and driving behavior, sharing of roads by motorized and non-motorized modes, overcrowding of vehicles, and inadequate traffic signals, signs, and traffic management (Pucher *et al.* [3]). These problems lead to high levels of congestion, traffic deaths and injuries and environmental pollution. In Kolkata (India), for example, the average speed during peak hours in the Central Business District (CBD) area is as low as 10 km/h (Singh [4]). In China, the Beijing-Tibet Expressway experienced the world's worst traffic jam ever, as traffic congestion stretched more than 100 km from August 14 to 26, 2010 (Hickman [5]). In road related crashes, fatality rates in china and India are 22 and 17 per 100,000 inhabitants, respectively which is higher compared to developed countries, 5 in the United Kingdom and 6 in Germany (Sivak and Schoettle [6]). In 2010, the Indian cities Chennai, Delhi, Mumbai and Kolkata, average annual particulate matter PM10 were measured at 55, 286, 97 and 136, respectively. This is 2.75 or higher times the WHO guideline (average annual particulate matter is 20 micrograms per cubic meter) indicating a truly alarming public health hazard [7].

In order to reduce congestion, the performance of the road system has to be improved through building new infrastructure and through improved operations of the existing infrastructure with efficient traffic control and management strategies. Design of useful traffic control and management measures is difficult and requires testing with various designs. In most cases, it is not practically feasible to carry out field tests of these designs. Microscopic simulation tools are commonly used to test different traffic management strategies because they mimic the driver behavior explicitly and in detail in a controlled environment. Driving behavior models, including both longitudinal and lateral movements of the vehicles, are key components of microscopic traffic simulation tools. The detailed level of vehicle movement in microscopic simulation models is needed to understand the underlying behavior at the formation of congestion and is necessary for evaluation the impact of various solutions on traffic flow.

Traffic flow characteristics in emerging and developing countries are substantially different from those in the developed countries, and so microscopic traffic models that are designed for homogenous traffic streams need to be adapted for these situations. This is exemplified by Figure 1. In homogenous traffic, vehicles are predominantly cars that follow the marked lanes. Traffic in emerging and developing countries is characterized by a wide mix of vehicles that includes motorized vehicles such as motorized two wheelers (MTW), auto-rickshaws (three-wheeled motorized vehicles), cars (including jeeps and small vans), buses, light commercial vehicles (LCVs), trucks, and non-motorized vehicles such as bicycles and tricycles. These vehicles have wide varying dimensions and performance capabilities. This variety leads vehicles not to follow strict lane discipline and occupy any available space on the road. Smaller size vehicles, such as two-wheelers, often utilize gaps between vehicles [8-9]. As a result, the interactions among vehicles and the resulting maneuvers they undertake are much more complex in mixed traffic conditions. Driving behavior models that describe these interactions are at the core of microscopic traffic simulation systems. Driving behavior models has been developed for more than half a century [10-12]. However, most of the available models are designed for homogenous traffic and so are not fully capable to reproduce traffic patterns that emerge in the presence of mixed traffic conditions. Reviews of these models in the context of homogeneous traffic can be found, for example, in Brackstone and McDonald [11], and Toledo [12].

This paper reviews the literature on driving behavior models that were specifically aimed for mixed traffic conditions. The models discussed include those for longitudinal and lateral movements and gap acceptance.

(a) Homogeneous Traffic (b) Mixed Traffic

Figure 1 Homogeneous and Mixed Traffic Characteristics

The rest of this paper is organized as follows: the next two sections reviews current models for longitudinal and lateral movements in mixed traffic. The following section discusses the limitations and gaps in state of the art models as well as data needs to support estimation of these models and improve their fidelity. The next section introduces new modeling framework and the final section summarizes our findings and conclusions.

2.0 LONGITUDINAL MOVEMENT MODELS

Longitudinal movement models commonly describe how a following vehicle reacts to the lead vehicle in the same lane. A large number of car following models have been proposed in the context of homogenous traffic. These may be classified based on behavioral assumptions, namely, stimulus response models [13-17] psycho-physical models [18-19] and fuzzy logic based models [20]. Car following models have also been extended to more general acceleration models that also consider free flow situations in which the subject driver does not closely follow a leader.

Studies of longitudinal movement for mixed traffic suggest extensions of the car following paradigm is several ways: First, drivers may react differently to their leader depending on the combination of the types of the two vehicles (their own and the leader). Second, the lack of lane discipline in the traffic stream causes drivers to react not only to their leader but also to other vehicles on their sides. Furthermore, the lack of lane discipline and the variability in vehicle widths result in situations in which drivers do not strictly follow a leader. For example, drivers may follow their leader only partially in a staggered way. They may follow two leaders at the same time or be squeezing between two leaders. Finally, in road sections with unseparated bidirectional flow, drivers may also respond to oncoming traffic sharing the same roadway.

2.1 Car following regimes

Lan and Chang [21] developed a car following model for motorcycles using the General Motors (GM) [13-14] model structure. They considered two cases: (1) only one leading vehicle in front; (2) two or more leading vehicles in front and neighboring-front (including either left-front, right-front, or both). In addition, an adaptive neuro-fuzzy inference system (ANFIS) was developed to capture the following behavior of motorcycle. They found that the ANFIS model performed better than the GM model.

Chakroborty *et al.* [22] proposed a longitudinal acceleration model that considers different driving behaviors in mixed traffic based on safety and urgency: free flow, car following, passing and presence of an opposing vehicle. The mathematical model is expressed by:

$$a_n\left(t+T_n\right)=\alpha\left[\beta(t)\left\{k_1\frac{V_s(t)-V_n(t)}{T_n}-k_2\dot{U}(t)\right\}\right]+(1-\alpha)\left[\frac{V_s(t)-V_n(t)}{T_n}\right] \tag{1}$$

Where, $a_n\left(t+T_n\right)$ is the acceleration/deceleration at time $t+T_n$. T_n is the perception/reaction time of the vehicle. $V_s(t)$ is the sustainable speed of the subject vehicle at time t. $V_n(t)$ is the subject's actual speed. $\dot{U}(t)$ is the rate of change of potential being faced by the subject. α is a state dummy variable that indicates whether or not the vehicle is constrained by dynamic obstacles (i.e., α is 0 for free flow and 1 for constrained flow). k_1 and k_2 are calibration constants. $\beta(t)$ is a sensitivity parameter.

The above equation depends on two terms: the deviation from the sustainable speed, which the driver feels comfortable driving at, and the rate of change of potential function. The potential function captures the magnitude of interaction with obstacles and other vehicles in the surroundings [23]. The interaction with obstacles depends on their characteristics. For example, the potential field due to a parked vehicle may be less pronounced than the potential field emanated by a truck coming in the opposing direction. The obstacles which are considered in the study are road edges, lane markings, static obstacles (e.g. potholes, parked vehicles) and dynamic obstacles (e.g. vehicles in the same and opposing directions).The results show that the model is capable to predict different driving regimes, from free flow to congested, in a single framework and to capture the effects of varying road geometry. However, the paper does not provide any details on the parameters' calibration.

Minh *et al.* [24] developed an acceleration model for motorcycles at signalized intersections. The model includes four driving regimes that are defined by combinations of car following or free flowing and acceleration or deceleration. Drivers are assigned to one of the four regimes based on the space headway to the leader. Free regime is invoked when distance headway is greater than the longitudinal threshold distance, otherwise the following-regime is applied. In free flow case, acceleration will be invoked when signal turns to green and deceleration regime is invoked when signal turns to red. In the following regime, the acceleration is invoked when relative speed is positive, otherwise deceleration regime is invoked. The acceleration in each regime is modeled using a generalization of the GM model [13-14] framework. The model takes into account the effect of the gender of the motorcycle driver, the number of people riding the motorcycle and whether the leader is a motorcycle or a four-wheeler:

Free acceleration: $$a_n\left(t+T_n\right)=\alpha\left[V_n^{DS}-V_n\left(t\right)\right]+\varepsilon_n\left(t+T_n\right) \tag{2}$$

Free deceleration: $$a_n\left(t+T_n\right)=\lambda\frac{\left(V_n\left(t\right)\right)^{\eta}}{\left(\Delta X_n\left(t\right)\right)^{v}}\upsilon_p^{\delta_n^p}\upsilon_g^{\delta_n^g}+\varepsilon_n\left(t+T_n\right) \tag{3}$$

Following acceleration: $$a_n\left(t+T_n\right)=\xi\frac{\left(V_n\left(t\right)\right)^{\varphi}}{\left(\Delta X_n\left(t\right)\right)^{\psi}}\left(\Delta V_n\left(t\right)\right)^{\lambda}\vartheta_p^{\delta_n^p}\vartheta_g^{\delta_n^g}\vartheta_h^{\delta_n^h}+\varepsilon_n\left(t+T_n\right) \tag{4}$$

Following deceleration: $$a_n\left(t+T_n\right)=\xi\frac{\left(V_n\left(t\right)\right)^{\varphi}}{\left(\Delta X_n\left(t\right)\right)^{\psi}}\vartheta_p^{\delta_n^p}\vartheta_g^{\delta_n^g}\vartheta_h^{\delta_n^h}+\varepsilon_n\left(t+T_n\right) \tag{5}$$

Where, $a_n\left(t+T_n\right)$ is the acceleration/deceleration of the subject motorcycle at time $t+T_n$. T_n is the subject's reaction time. $V_n(t\ )$ and V_n^{DS} are the speed of the subject and its desired speed, respectively. $\Delta X_n(t)$ is the spacing between the subject and the leader or the stop line. λ, α, ξ, ϕ and ψ are parameters. δ_n^p, δ_n^g and δ_n^h are the dummy variables associated with multiple riders on the motorcycle, the gender of the driver and four wheeler leaders, respectively. υ_p, υ_g, ϑ_p, ϑ_g and ϑ_h are the parameters associated with these dummy variable. $\varepsilon_n\left(t+T_n\right)$ is a random error term.

The parameters of all components of the model were estimated jointly using the maximum likelihood method with trajectory data of individual vehicles. However, only 20 trajectory data points at a resolution of 0.2 seconds (thus covering only 4 seconds of travel) were available due to limited field of view. The results showed that accelerations and decelerations were larger in absolute values when the driver was alone on the motorcycle and when the lead vehicle is a four wheeler. Accelerations and decelerations were lesser for female drivers compared to male drivers.

Ravishankar and Mathew [25] included vehicle-type specific parameters for different combinations of leaders and followers in the Gipps's car-following model [15]. They studied all nine combinations of leaders and followers consisting of auto-rickshaws, cars and buses. They introduced type-specific parameters for the maximum comfortable acceleration and for the desired speed in the acceleration model and for the maximum deceleration in the deceleration model. In addition, different parameters for the sensitivity of the deceleration to the spacing between the vehicles were introduced for each leader-follower combination. The model parameters were estimated using trajectory data collected with GPS devices that were installed in pairs of vehicles that participated in following experiments. The estimation results showed that the smaller Auto-rickshaws-tend to maintain lesser space headways compared to larger vehicles.

2.2 Other following regimes

Several studies acknowledged that, in mixed traffic, drivers may not strictly follow their leader, but only be partially aligned with it, following multiple leaders or being between leaders. Cho and Wu [26] developed a model based on the concept of thrust and repulsion. In their model, the speed of the subject motorcycle is a function of its desired speed and current speed, the current speed of the leader, the space headway and a minimum safe headway:

$$V_n(t+1) = V_n^{DS}\left(1-\exp\left(-\lambda\frac{\left(V_{n-1}(t)\right)^{\alpha}}{\left(V_n(t)\right)^{\beta}}\left(\frac{\Delta x_n(t)-S_{n-1}}{L}\right)^{\gamma}\right)\right) \tag{6}$$

Where, $V_n(t)$ and $V_{n-1}(t)$ are the speeds of the follower (subject) and the leader at time t, respectively. . V_n^{DS} is the desired speed of the follower. $\Delta x_n(t)$ is the space headway between the leader and the follower. S_{n-1} is the minimum space headway at a standstill. α, β, γ, λ and L are parameters.

In order to study the staggered following, a weight function that captures the lateral separation between the subject and leaders was introduced in the model. The model allows the possibility of two leader motorcycles (the nearest on the right-hand and left-hand sides). The speed of the subject vehicle is calculated using the following equation:

$$V_n(t+1) = V_n^{DS}\left(\begin{array}{l} 1-w(y_r(t)-y_n(t))e^{-\lambda\frac{(V_r(t))^{\alpha}}{(V_n(t))^{\beta}}\left(\frac{x_r(t)-x_n(t)-S_r}{L}\right)^{\gamma}} \\ -w(y_l(t)-y_n(t))e^{-\lambda\frac{(V_l(t))^{\alpha}}{(V_n(t))^{\beta}}\left(\frac{x_l(t)-x_n(t)-S_l}{L}\right)^{\gamma}} \end{array}\right) \tag{7}$$

Where, w is the weight function. $y_r(t)$ and $y_l(t)$ are the lateral positions of the nearest lead vehicle on the right-hand and left-hand sides, respectively. $y_n(t)$ is the subject's lateral position.

This study considers multiple leaders and staggered following in the longitudinal behavior models. The limitation is that calibration and validation of the model from field data is not reported.

Lee *et al.* [27] developed a car following model for motorcycles that requires the driver to maintain a minimum gaps that would allow it to stop in time to avoid a collision with the leader if it breaks to a stop. But, because motorcycles may be staggered with their leaders and can easily maneuver laterally, the model also allows them to keep shorter following distances if these allow them to dodge the crash by moving laterally, as shown in Figure 2. The minimum distances are calculated in both cases based on equations of motion under constant decelerations:

$$S_n^{\min}(t) = \min\left\{\begin{aligned}&T_n - \frac{V_n^2(t)}{2b_n} + \frac{V_{n-1}^2(t)}{2b_{n-1}}, \\ &\Delta V_n(t)\left(T + \frac{d_n(t)}{v_n}\right) + \frac{1}{2}(b_n' - b_{n-1})\left(\frac{d_n(t)}{v_n}\right)^2 - \frac{1}{2}b_{n-1}T\left(T + \frac{2d_n(t)}{v_n}\right)\end{aligned}\right\} \tag{8}$$

Where $S_n^{\min}(t)$ is the minimum following gap. $V_{n-1}(t)$ and $V_n(t)$ are speeds of the leader and follower, respectively. $\Delta V_n(t)$ is the speed difference between the two (speed of the subject less the speed of the leader). b_{n-1} and b_n are their decelerations when braking to a stop. b_n' is the deceleration of the subject motorcycle when moving laterally. T_n is the reaction time. $d_n(t)$ is the lateral movement needed by the subject in order not to overlap laterally with the leader. v_n is the lateral speed.

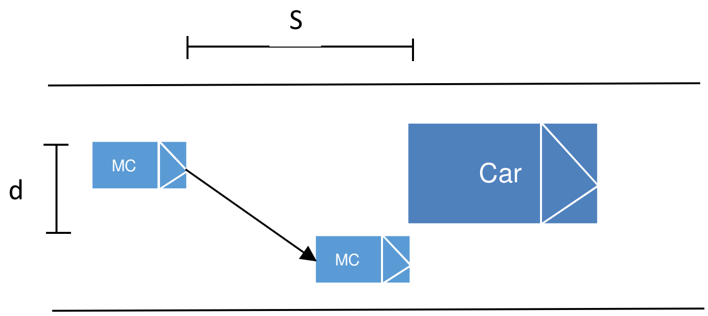


Figure 2 Lateral Movement for Collision Avoidance (adopted from [27])

The authors also introduced a model for the minimum following gap in oblique following for cases that the subject does not laterally overlap with the leader, as shown in Figure 3. The model assumes that there are minimum longitudinal and lateral gaps that the subject would maintain if strictly behind or parallel to the leader. When the subject is oblique to the leader, the minimum space gap would be defined by a linear interpolation of these two values. The minimum longitudinal and lateral gaps are given by:

$$S_n^{long}(t) = S_o^{long} + \alpha_1^{long}\Delta V_n(t) + \alpha_2^{long}V_{n-1}(t) \tag{9}$$

$$S_n^{lat}(t) = S_0^{lat} + \alpha_1^{lat}\Delta V_n(t) \tag{10}$$

It is assumed that vehicles follow elliptical path in oblique following behavior. The equation of elliptical curve is described as follows:

$$S_n^{oblique}(t) = \frac{S_n^{long}(t)\, S_n^{lat}(t)}{S_n^{long}(t)\sin\theta \cdot S_n^{lat}(t)\cos\theta} \tag{11}$$

Where, $S_n^{long}(t)$, $S_n^{lat}(t)$ and $S_n^{oblique}(t)$ are the minimum longitudinal, lateral and oblique gaps, respectively. θ is the following angle. S_o^{long}, α_1^{long}, α_2^{long}, S_0^{lat} and α_1^{lat} are parameters.

The more constraining minimum longitudinal distance among $S_n^{\min}(t)$ and $S_n^{oblique}(t)$ is used as the safety margin in the calculation of accelerations using Gipps' model [15]. The longitudinal headway model and the oblique and lateral headway models were calibrated by using detailed vehicle trajectory data.

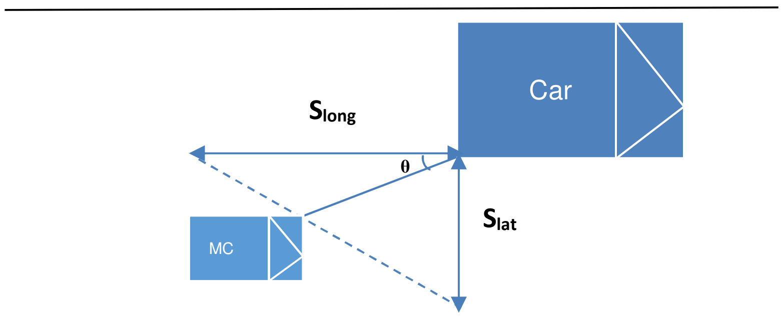


Figure 3 Oblique Following (adopted from [27])

Jin *et al.* [28] proposed a modification to the optimal velocity model [19] to capture staggered car-following situations (Figure 4). The modified model takes into consideration the extent of lateral overlap between the leader and follower with additional time to collision .The mathematically model for acceleration is given as follows:

$$a_n(t+T_n)=\alpha\left[V^{opt}(\theta_n(t))-V_n(t)\right]-\frac{\lambda}{TTC_n(t)} \tag{12}$$

Where, $a_n(t+T_n)$ is the acceleration of the subject vehicle, $TTC_n(t)$ is the time to collision. α and λ are parameters. $V^{opt}(\theta_n(t))$ is the subject's optimal velocity, which depends on the width of the leader and visual angle and can be written as:

$$V^{opt}(\theta_n(t))=V_1+V_2\tanh\left\{C_1[w_{n-1}/\theta_n(t)]-C_2\right\} \tag{13}$$

Where, V_1 = 6.75 m/s, V_2 = 7.91 m/s, C_1 = 0.13 m−1, and C_2 =1.57 are parameters that were obtained in a previous study [29].

The TTC variable takes into account lateral separation effects. It can be expressed as:

$$\frac{1}{TTC_n(t)}=\frac{\theta_n^{'}(t)}{\theta_n(t)}-\frac{\varphi_n^{'}(t)}{\varphi_n(t)} \tag{14}$$

$$\theta_n(t)=\frac{w_{n-1}}{\Delta x_n(t)-l_{n-1}} \tag{15}$$

$$\varphi_n(t)=\arctan\frac{b_n}{\Delta x_n(t)-l_{n-1}} \tag{16}$$

Where, $\theta_n(t)$ is the visual angle which is observed by the driver of the n^{th} vehicle at time t. $\varphi_n(t)$ is the visual gap angle separating the leader from the moving direction of the subject. $\Delta x_n(t)$ is the distance headway between the leader and subject. b_n is the lateral separation distance between the two vehicles. l_{n-1} and w_{n-1} are the length and width of the leader, respectively.

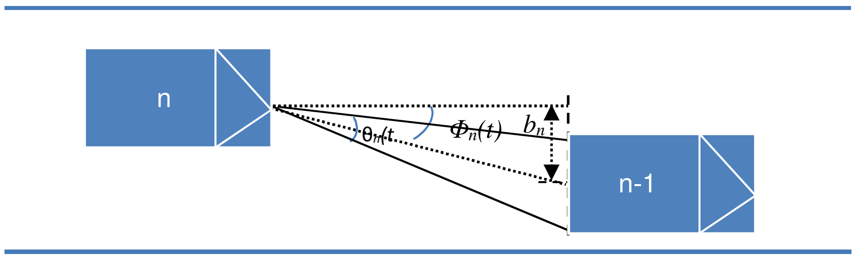


Figure 4 Staggered Car Following (adopted from [28])

The authors conducted stability analysis of the proposed model. However, they did not estimate or validate their model with real-world data.

Gunay [30] proposed a car following model that considers the lateral friction with surrounding vehicles. In this model, the subject vehicle chooses a maximum speed that, in order to avoid crashing with a leader that brakes to a stop, would allow it to either squeeze between two leaders or to shift laterally from the path of the leader. Squeezing between two leaders can occur at a Maximum Escape Speed (MES), which depends on the lateral clearance between the leaders (Figure 5):

$$MES = -17.2(FC)^2 + 77.6(FC) - 0.7 \qquad 0.5 < FC < 1.5 \tag{17}$$

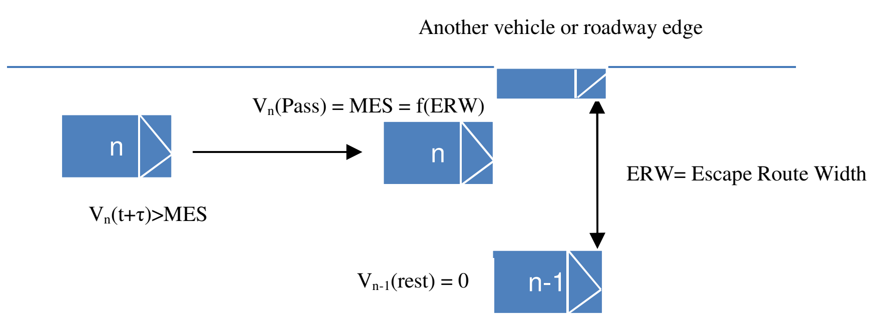


Figure 5 Speed to Allow Squeeze Pass the Leader (adapted from [30])

The maximum speed that would allow the driver to undertake the squeezing maneuver is given by Gipps' model framework [15]:

$$V_n(t+T_n) \le b_n T_n + \sqrt{(b_n T_n)^2 - 2b_n \left\{ V_n(t)\frac{T_n}{2} + \frac{MES^2}{2b_n} + \frac{V_{n-1}^2(t)}{2b_{n-1}} + x_n(t) - x_{n-1}(t) + S_{n-1} \right\}} \tag{18}$$

Where, $V_n(t)$ and $V_{n-1}(t)$ are the speeds of the subject and the leader, respectively. T_n is the subject's reaction time. b_n and b_{n-1} are the deceleration rates of the subject and the leader, respectively. $x_n(t)$ and $x_{n-1}(t)$ are the positions of the subject and leader, respectively. S_{n-1} is the length of the leader vehicle.

At the same time, the driver should also be able to veer laterally to avoid crashing with the leader, as shown in Figure 6. The maximum speed that would allow the subject to veer and avoid a crash with the leader if it comes to a stop is given by:

$$V_n\left(t+T_n\right) \le 2\frac{x_{n-1}(rest)-x_n(t)-\frac{V_n(t)}{2}T_n-\frac{t_{veer}}{2}MES-d_{body}}{t_{veer}+T_n} \tag{19}$$

Where, $x_{n-1}(rest)$ is the position of the leader after coming to a stop. t_{veer} is the time taken for the veering manoeuver, which depends on the veering distance. d_{body} is the distance between the center of bodies of the two vehicles, at the time that the passing takes place.

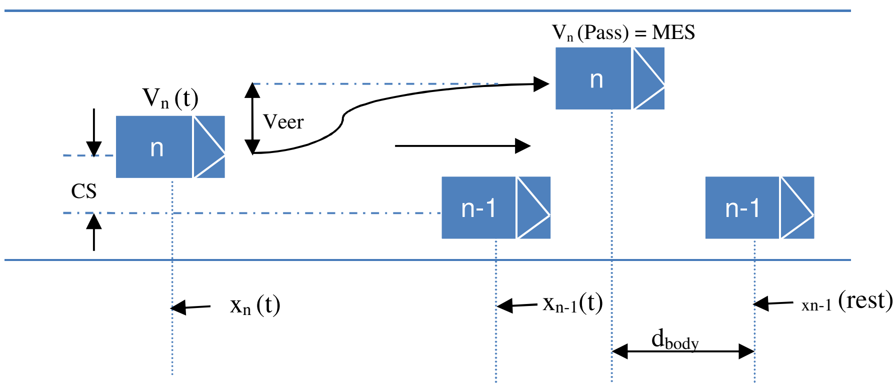


Figure 6 Speed to Allow Partial Lane Change (adapted from [30])

The study presents this theoretical framework, but does not make any attempt to estimate the model parameters with field data.

In summary, several authors have proposed modifications and variants of strict car following models that have also been used in modeling homogeneous traffic that capture the differences between various vehicle types that are present in the mixed traffic stream. Others, have suggested models for non-strict following situations, such as staggered following [26, 27, 28, 30] and passing behavior [30]. Only limited research has been done to integrate these various regimes in a unified following model framework. In terms of data and estimation, some of the proposed models [e.g. 22, 27, 28, 30] require difficult to collect data, such as on visual angles, escape and veering speeds. Minh *et al.* [24], Lee *et al.* [27] and Ravishankar and Mathew [25] used trajectory data for estimation and validation of their models. The lack of such data dictated that other studies relied on macroscopic data for the model calibration and validation.

3.0 LATERAL MOVEMENT MODELS

Lane changing models describe the dynamics of lateral movement behavior of vehicles. They incorporate the decision to initiate a lane change and its execution. The distinction between the wish to change lanes and the execution of the lane change was introduced by Sparmann [31]. Lane changing may be mandatory (MLC) or Discretionary (DLC). MLC lane change are those that the driver must take, for example in order to turn at an intersection or avoid obstacles. DLC are motivated by the drivers' desire to improve their current driving conditions by overtaking a slow vehicle or having a shorter queue. This structure was implemented in CORSIM [36].

Lane changing models are often based on decision rules (Gipps [32], SITRAS [33-34], Wei et al. [35]). In this approach, drivers select lanes by comparing the acceptable lanes with respect to, a hierarchy of considerations, such as downstream lane blockages, lane use restrictions, the locations of obstructions, the presence of heavy vehicles, and potential speed gains. Other studies (e.g. Yang [37], Ahmed [38] and Toledo [39]) used the random utility theory, which captures trade-offs among the various considerations, to describe the lane selection behavior. These models are commonly estimated using the maximum likelihood approach based on vehicular trajectory data.

3.1 Lateral shift behavior under mixed traffic conditions

Conventional lane-changing models are designed for lane-based movements. They cannot describe the lateral movements of mixed traffic adequately. Due to non-lane discipline and smaller size of vehicles, lateral movements occur also without changing lanes entirely. The following studies describe lateral movement behavior for mixed traffic.

Malikarjuna *et al.* [40] studied the lateral gap maintaining behavior in heterogeneous traffic conditions. In this study, the data was extracted using video image processing software, TRAZER. The data was collected for different road widths ranging from 6.60 m to 12.50 m. Four vehicle combinations were considered such as light motorized vehicle (LMV)-Two-wheeler (TW), TW-LMV, LMV-LMV, TW-TW. Lateral gaps from left side and right side of the subject vehicle were considered. The following factors were considered in this study: speeds and types of subject vehicle and adjacent vehicles. The results of the study are that if the speed of the adjacent vehicle increases, lateral gap between subject vehicle and adjacent vehicle increase. This study focuses only on empirical results and does not deal with any driver behavior model.

Luo *et al.* [41] studied the interaction between cars and bicycles in heterogeneous traffic conditions. They developed a cellular automata (CA) model with an occupancy rule based on lateral gap of mixed traffic. Bicycles move laterally within the bicycle lane or to the cars lane, with different required gaps. Using traffic video data that were collected in Beijing, China, they developed a regression model to find needed lateral gaps based on speed of the car. The study results show that lateral gaps increase with an increase in speed. The study set up was limited to a situation that cars and bicycles move in fixed lane for each type.

Cho and Wu [26] proposed a lateral movement model for motorcycles that assumes drivers try to modify their positions to get the maximum lateral space and the motivation decides the lateral movement. Lateral position of a motorcycle (Vehicle N) was decided by the positions of the nearest vehicles at front left (vehicle J), front right (vehicle I), adjacent left (Vehicle L), adjacent right (Vehicle R), rear left (Vehicle Q), and rear right (Vehicle P) as shown in Figure 7. The following factors which affect the lateral movement model are longitudinal and lateral position of surrounding vehicles, vehicle performance and maximum steering angle. The vehicle will modify its lateral position to the middle of those vehicles surrounding it.

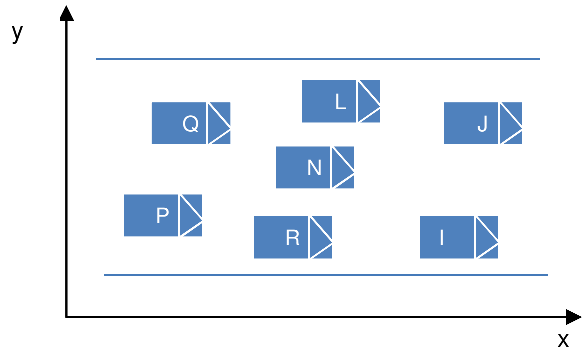


Figure 7 Vehicles on a Motorcycle Lane - Lateral Movement (adopted from [26])

The lateral movement model developed by Chakroborty *et al.* [22] uses the maximum steering angle and speed of the vehicle to define a set of accessible points for the subject vehicle. Among these points, the driver chooses the one that has the least interaction with other vehicles and obstacles. Calibration of the model parameters was not discussed.

Oketch [42] presents a model for mixed-traffic streams with motorized and non-motorized vehicles. The lateral movement decisions are governed by fuzzy logic rules. The decision process of lateral movement is modeled using three steps: 1) identification of options 2) their evaluation by fuzzy logic and 3) testing the safety criteria (available gaps) before execution of the actual manoeuver. The lateral movement evaluation considers avoiding obstructions, directional movement requirements, avoiding slow moving vehicles and gaining speed and queue advantage. The model incorporates gradual lane change manoeuver (as opposed to an instantaneous one), by assuming lateral speed of 1.0 m/sec for each vehicle. Validation and calibration of the model were carried out with macroscopic data (delay, queue lengths, mid-link speeds and link travel times) from Nairobi, Kenya.

Mathew *et al.* [43] proposed a modeling framework using the concepts of strips to capture the lateral movements. The benefit from changing strips stems from the difference between the safe speeds on the two strips as computed using the car-following model. In order to represent tactical lateral movement, the driver evaluates multiple strip changes. The benefit of each strip depends on the speed advantage and decays with the number of required strip changes:

$$b_{s_n(t)} = \frac{v_{safe}(t, s_n) - v_{safe}(t, s_c)}{v_{\max, s_c}} \times e^{-\lambda \times s} \qquad (20)$$

Where, $b_{s_n(t)}$ is the benefit of changing to strip s_n. s_c is the current vehicle's strips. v_{safe} and $v_{\max}$ are the safe speed and the maximum speed in the strip. s is the number of strip changes to strip s_n. λ is a parameter.

The model was validated using macroscopic data (throughput, average speed and travel time) that were collected in Mumbai, India.

Some of the studies model the lateral movement behavior of vehicles using discrete choice models. For example, Lee *et al.* [27] developed a model for lateral movements of motorcycles using path choice model. Such path choice behavior is described by using a Multinomial Logit model. There are three alternatives in the choice set: shifting leftward, keeping straight, and shifting rightward. These alternatives are formulated based on the speed of the vehicle in front, interacting force with the front and rear vehicles, size of the vehicle near the path, lateral distance to the ready to overtaken position, lateral clearance beside the preceding vehicle. The lateral movement distance for the next time step was calculated based on lateral speed of the vehicle. The models were calibrated on the basis of trajectories of motorcycles recorded at a section of the Victoria Embankment in central London. But in this study, only lateral movement behavior of motorcycles was studied.

Siddique [44] developed discretionary and mandatory lateral movement models under week lane-discipline conditions using a multinomial logit (MNL) model. The road was divided into a number of strips with a width of 0.5m, which formed the discrete alternatives. The variables considered in the model included the subject vehicle type, speed, lead vehicle type and follower vehicle type, position of the road, type of movement and mandatory critical zones. It was estimated with trajectory data that were collected from two locations of Dhaka, Bangladesh. The results show that non-motorized slow-moving vehicles prefer to stay on the left (slow) side of the roadway whereas other vehicles tend to move on the right (fast) side with an expectation to gain speed. A limitation of the data used in the study is that due to limited field of view of the cameras used to collect the data, it was not possible to record the movement of the vehicle for a long distance.

Munigety *et al.* [45] presents a lateral movement model for different vehicle types such as motorcycles, cars, auto-rickshaws and heavy vehicles using discrete choice analysis. The framework of the lateral-shift decision model is described using a Multinomial Logit model. There are three alternatives in the choice set: shifting leftward, keeping straight, and shifting rightward. The explanatory variables of the model are speed of the vehicle ahead, gap and size of the vehicle in front. These models are estimated using detailed vehicle trajectory data that was collected in mixed traffic driving conditions. The output of the study in the context of speed gain is that cars and two-wheelers preferred faster path whereas; heavy vehicles and three-wheelers preferred slower path. This implies that heavy vehicles and three-wheelers may change their current path in order to prevent obstructing the fast moving vehicles which approaching from the rear. The longitudinal gap turned out to be an insignificant variable for two-wheelers in the context of space gap. This may be due to its smaller size which allows it to enter any convenient path once it finds a sufficient lateral gap.

In summary, Oketch [42] has dealt with mandatory lateral shift and other studies have dealt with discretionary lateral shift. Several authors proposed discrete lane change models that are similar to those used with homogeneous traffic conditions. Oketch [42], Mathew et al. [43] and Siddique [44] applied these concepts on a finer scale by dividing the roadway into a number of narrower strips (corresponding to the width of a motorcycle). The vehicle moves laterally discretely between these strips. Continuous lateral movement models may provide a more realistic description of this behavior. Lee et al. [27], Siddique [44] and Munigety et al. [45] used trajectory data to estimating the direction of lateral shift through discrete choice models. The lack of such data dictated that other studies were not estimated with real-world data.

3.2 Gap acceptance models

Gap acceptance models describe whether there is a sufficient gap present for the vehicle to execute the desired lane change or shift manoeuver. In these models, the driver compares the available gap between the vehicles in the desired lane with a corresponding critical gap. , the driver will invoke lane change if the available gap is larger than the critical gap. Critical gaps are modeled as random variables using various distributions in to capture its variability across drivers. Drew *et al.* [46], Cohen *et al.* [47] and Solberg *et al.* [48] used the lognormal distribution to describe critical gap. Miller [49] assumed it to be normally distributed. The influence of different traffic factors on critical gaps was discussed in several studies [50-53]. Ahmed [38] allowed different sets of parameters for MLC and DLC situations. Choudhury *et al.* [54] and Hidas [55] distinguished between normal and forced lane changing, in which the subject vehicle forces the lag vehicle to decelerate.

Most studies related to gap acceptance model in the context of mixed traffic deal with yield controlled intersection crossing behavior. They mostly use constant critical gaps that differ between various categories of vehicles (e.g. Popat *et al.* [56], Raghavachari *et al.* [57]). Similarly, Agarwal *et al.* [58] used different constant critical gap values for trucks/buses, cars/two-wheelers, auto-rickshaw, and cycles. Kumar and Rao [59] distinguished between critical gaps on the near and far lanes at the intersection.

Sangole *et al.* [60] used Neuro-Fuzzy technique to model the gap acceptance behavior of right turning two-wheelers at three-legged intersections. Data for this study were collected at two three-legged right angled intersections in Aurangabad, India. The gap acceptance decisions depend on the size of lag/gap, age of the driver, conflicting vehicle type, and the vehicle occupancy. The study found that two-wheelers accept gaps as short as 1 second. Large gaps over 9.5 seconds were accepted in all cases.

Several studies incorporate the variations in the critical gap by assuming that they follow some probability density function. For example Hossain [61], within the MIXNETSIM model for roundabouts, used a lognormal distribution model following analysis of field data from Dhaka, Bangladesh. In the simulation model, each driver is assigned different critical lags/gaps from this distribution. The model was validated using data on the travel times of vehicles through the roundabouts and macroscopic relationships of the flows circulating flows.

Pawar and Patil [62] analyzed gaps and lags at four-legged partially controlled intersections in India. They estimated critical gaps using several different estimation methods. Critical lags and gaps vary and depend on the subject vehicle type, speed, position of the conflicting vehicle. Depending on the method of estimation temporal were estimated between 2.8 seconds and 3.9 seconds, ans spatial critical gaps were between 31.8 m and 36 m. These values are smaller than similar values reported in developed countries indicating drivers' aggressiveness in India.

Ashalatha and Chandra [63] proposed an alternative definition of critical gaps using the clearing behavior of vehicles. They defined a rectangular conflict zone which has the width of the lane and a length which is related to the length of the crossing vehicle (Figure 8). It is assumed that the critical gap is the time needed for the crossing vehicle to clear this area. Estimation results using this definition yielded critical gaps that were lower than those given in HCM but greater than those estimated by standard critical gap estimation methods.

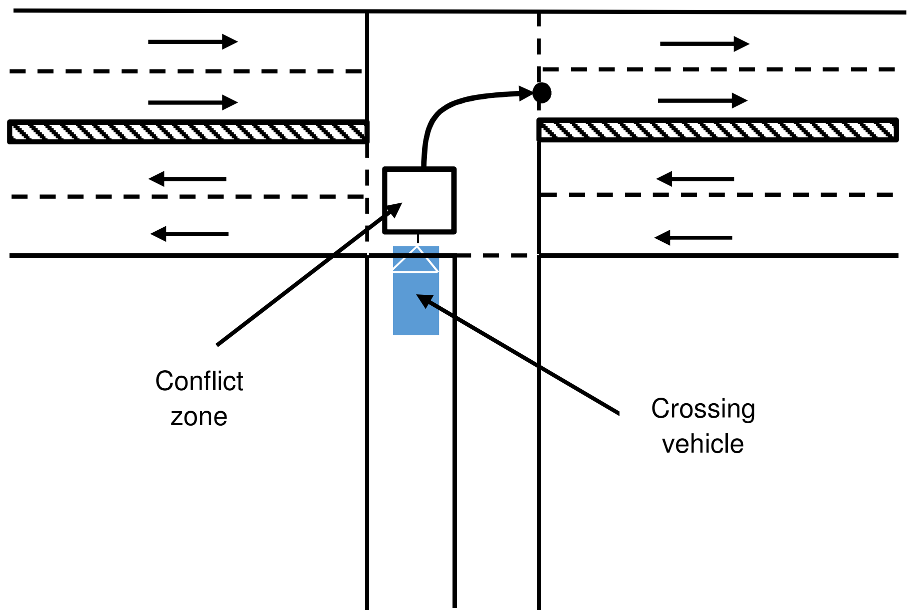


Figure 8 Schematic Representation of Conflict Zone (adapted from [63])

Kanagaraj *et al.* [64] studied merging manoeuvers at T-junctions under congested traffic conditions. They developed probabilistic merging models. This is one of the first attempts to investigate merging behavior under mixed traffic conditions. The critical gap functional form was expressed as follows:

$$G_{cr\ \mathrm{n}}^{M}(t) = \exp\left[\beta^{M} X_{n}(t)\right] + \varepsilon_{n}^{M}(t) \tag{21}$$

Where, $G_{cr\ \mathrm{n}}^{M}(t)$ is the critical gap for merging. $X_n(t)$ and β^M are the vector of explanatory variables that affect critical gaps and the corresponding parameters, respectively. $\varepsilon_{n}^{M}(t)$ is anormally distributed random term.

The explanatory variables used in the model included the lead, lag and subject vehicle type, the speeds of the lead and lag vehicles, the subject's waiting time and the traffic volume on the main road. The model was calibrated and validated using field data collected in Chennai, India. The results showed that the critical gaps for smaller vehicles are smaller than those for cars.

Kanagaraj *et al.* [65] studied two unique merging processes which are commonly observed in mixed traffic: group and vehicle cover merging. Probabilistic models for these behaviors were developed. In group merging several vehicles merge in the same gap at the same time. Critical gaps for this case depended on the lead and lag vehicle speeds, the gap between the lead and lag vehicle, the number of vehicles in the group and the time it has been waiting to merge. Vehicle cover merging describes a situation that a vehicle merges under the cover of another (often larger) vehicle that interferes with the lag vehicle. Critical gaps in this situation depend on the lateral gap longitudinal and lateral gaps, the lead vehicle type and the subjects' waiting time.

The models were estimated and validated using field data collected in Chennai city, India. Two-wheelers were found to be more likely to accept use these merging behaviors compared to auto-rickshaw. Vehicles tended to accept smaller gaps when the lead vehicle is a two-wheelers compared to cars and auto-rickshaws. Similarly, two-wheeler tended to reject gaps more when the lag vehicle is a car.

In summary, gap acceptance studies in the context of mixed traffic focus on intersection crossing and merging behavior. They do not describe the lateral shift process in mid-block sections. The developed models generally adopt the gap acceptance framework used in the context of homogeneous traffic conditions, but with additional explanatory variables, mostly in order to capture differences in behavior among different vehicle types. As with homogeneous traffic models, in congested conditions, cooperative and forced lateral shifts may also take place. These have been modelled by Kanagaraj *et al.* [64]. Hence, in this context as well, there is a need for a unified model that describes lateral movement including both lateral shift and gap acceptance behavior.

4.0 CHALLENGES AND RESEARCH DIRECTIONS

There are two important characteristics that distinguish mixed traffic flow from homogenous traffic: the presence of a mix of widely variable vehicle types, and organization of lane-less traffic. The mix of vehicles can be captured by developing type-specific driving behavior models. For example, Lee *et al.* [27] developed behavior models specifically for motorcycles. Alternatively, type-specific parameters may be added to generic driving models. For example, Asaithambi *et al.* [66] and Mathew and Ravishankar [67] simulated mixed traffic using vehicle type-specific parameters in car following models. The non-lane based movements of vehicles have also been studied in the literature. The lateral movement of vehicles results, especially for two-wheelers that the leader-follower pair changes frequently. Figure 9 shows the distribution of duration that various vehicle types are behind the same leader [68]. The figure shows that following episodes tend to be very short. 80% of two-wheeler will follow the same leader for less than 3 seconds, whereas only 70% and 66% of auto-rickshaw and larger size vehicle. This may be due to motorcycle's smaller size and better maneuverability. At the other end of the distribution, 15% of larger size vehicles and 11% of auto-rickshaws follow the same leader for more than 8 seconds. But, only 5% motorcycles experience similar following episodes. The results imply significant lateral movements in the traffic stream and suggest that the two-dimensional movement of vehicles need to be integrated in a comprehensive driver behavior model. To this end, several research directions to advance driver behavior models for mixed traffic flows, through improved modeling, data collection and model estimation, are discussed next.

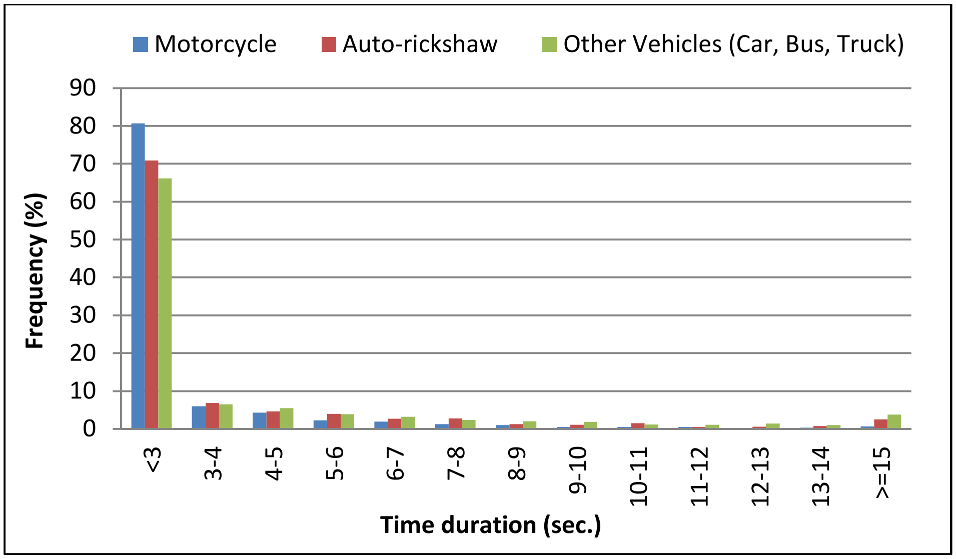


Figure 9 Frequency of Same Leader Present for Different Types of Following Vehicle

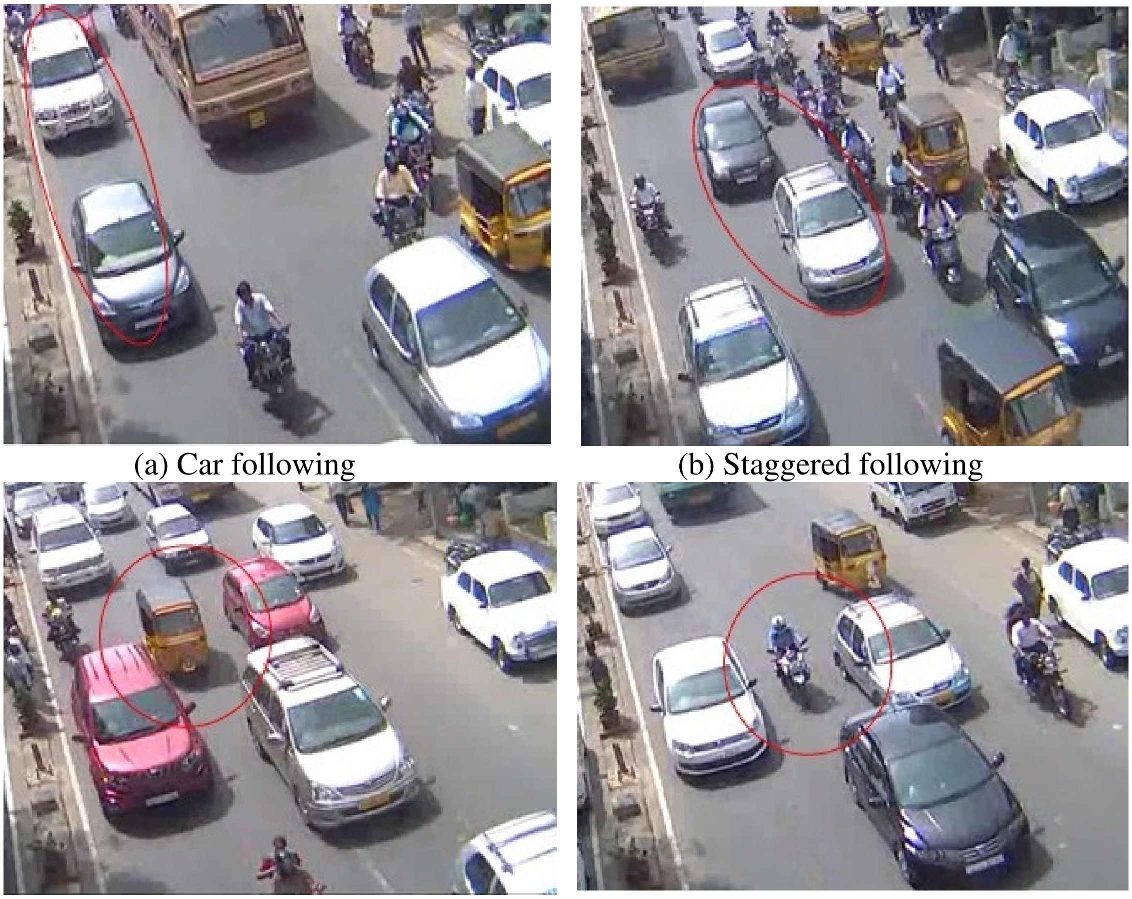
4.1 Driver Behavior Models

Some directions for improvement of the specification of state-of-the-art models are as follows:

1. <u>Driving regimes</u>: A wider range of driving regimes exists in mixed traffic streams. For example, in the longitudinal movement, drivers may not only strictly car follow, but also have different interaction regimes with their leaders. These regimes needs to first be clearly defined, for example, using the extent of lateral overlap or gap with the leader as a classifying variable. Figure 10 shows examples of various following regimes:

- *Car Following*: In this case, the lag vehicle (car) strictly following with leader (car).
- *Staggered Following*: Due to lane-less traffic and different type of vehicles, the following vehicle (car) is staggered with the leader vehicle (car), which implies looser following, and a better field of view and opportunities to initiate lateral shifts.
- *Following between two vehicles*: In mixed traffic, vehicles occupy any lateral position on roadway, based on space availability. Hence, the subject vehicle (auto-rickshaw) follows between two leaders (car and car). This also allows better field of view and opportunities to pass between two vehicles or initiate lateral shifts.
- *Passing*: Passing is a typical behavior of two-wheelers in mixed traffic due to their narrow size and high maneuverability. The existence of this behavior pattern has been pointed out in several studies [42, 69-72].

2. <u>Multiple leaders</u>: Due to non-lane based movement and different vehicle sizes, a vehicle may follow multiple leaders. Following models need to identify the lead vehicle that affects the subject vehicle movement to a greater extent. To the best of our knowledge, the effect of multiple leaders has not been incorporated in the existing models.



(a) Car following (b) Staggered following

(c) Following between two vehicles (d) Passing

Figure 10 Following Behaviors in Mixed Traffic

3. Adjacent vehicles: In homogenous traffic following models, the subject vehicle considers only the leader in the same lane. It is commonly assumed that adjacent vehicles in other lanes do not affect the longitudinal movement. This assumption is less reasonable in in mixed traffic. Vehicle's movement may be affected by adjacent vehicles. For example in the passing situation (Figure 10d), the speeds of the passing vehicle may be affected by the lateral gap between the adjacent vehicles. The effect of adjacent vehicles was proposed by Gunay [30], but is absent in most other studies.

4. Lateral movement: Lateral movement is often modeled in discrete lanes or strips. Several authors (Oketch *et al.* [42], Arasan and Koshy [9], Kanagaraj *et al.* [73], Asaithambi *et al.* [74]) assumed a constant lateral speed (e.g. 1 m/sec). Siddique [44] assumed move discretely between strips (each 0.5 m wide). Mathew *et al.* [43] also assumed discrete lateral movement allowing vehicles to move only one strip at a time. The different vehicles types in the traffic stream vary widely in their dynamic capability, which affects their lateral movement. For example, motorcycle can move laterally much faster compared to larger size vehicles. Hence, lateral movement models may be extended to account for these differences in speed and maneuverability.

5. Lateral shift processes: Due to the non-lane based conditions, lateral shifts occur frequently in mixed traffic streams, and therefore need to be modeled in detail. As with homogenous flow, these can take place using in different processes, such as normal, forced and cooperative lateral shifts.

These also differ in the effect on other vehicles and traffic flow. Kanagaraj *et al.* [64] studied normal and forced merging behavior at intersections. However, more research is needed on these shift mechanisms and in more general settings.

6. Desired lateral positioning: Several authors (e.g. [44], [68]) showed that drivers have preferred lateral positions in different situations. For example, cars tend to prefer the far side of the roadway that offers higher speeds and lesser friction with other vehicle types and obstructions (e.g. parked vehicles, bicycles and pedestrians). Motorcycles and auto-rickshaws tend to keep to the near side. Driver behavior models should be developed that capture the lateral position preferences.

4.2 Model Estimation and trajectory data

Driving models in homogenous traffic have been estimated with econometric methods and using detailed trajectory data (e.g. Toledo *et al.* [75], Choudhry [76]) However, in the context of mixed traffic calibration and validation have mostly been based on macroscopic flow characteristics, such as flows, speeds and densities [8-9, 66, 77]. This approach limits the level of detail that can be captured in the developed models. Few studies utilized trajectory data, but these are often small samples collected for a specific study and for limited field of view [27, 64, 78].

Trajectory data are obtained using the video recordings [38, 79-80] and naturalistic studies [81]. FHWA'S Next Generation Simulation project [82] shared several datasets of vehicle trajectories collected on expressway and urban arterials in US. These have been used extensively to calibrate and validate driving behavior models [83-87, among others] for homogeneous traffic. To the best of our knowledge, limited vehicle trajectory data are available in the context of mixed traffic. This may, to a large extent, be due to the difficulty and high cost involved in data collection and extraction, and the technical complexities associated with having a wide mix of vehicles types with varying physical dimensions and dynamics characteristics (speed and acceleration capabilities) and non-lane based movement. Few studies involved collection of mixed traffic trajectory data. Lee *et al.* [27] extracted trajectories of 2019 motorcycle and other vehicles on an 80 meters section in London. Mallikarjuna *et al.* [88] developed TRAZER an automated image processing system to extract trajectories from video records. They collected six hours of data from a 25 meters section of a road in Delhi, India. Munigety *et al.* [45] collected trajectories of 3173 vehicles on a 320 m road section in Mumbai, India. A recently collected data set that was collected by Kanagaraj et al. [68] is available as open source at http://toledo.net.technion.ac.il/mixed-traffic-trajectory-data/. This dataset includes 3005 vehicle trajectories at a resolution of 0.5 seconds on a 200 meters section in Chennai, India. In all these studies, the trajectory length is short due to the limited field of view of the cameras. Observations on longer sections are necessary in order to model complex behaviors patterns.

5.0 MODELLING FRAMEWORK OUTLINE

This section outlines an integrated model for driving behavior that captures both longitudinal and lateral movement of the vehicles under mixed traffic conditions. Figure 11 shows the overall framework. In the first step, the drivers' goal is defined in terms of a desired manoeuver. Based on the literature, types of desired manoeuvers that may be considered include Car following (CF), Staggered following (SF), Following between two vehicles (FB), and Passing (PS). Examples of these situations were shown above in Figure 10. The choice among the various alternatives may be based on decision rules or discrete choice models, and affected by the neighboring vehicles and their relative locations and speeds, the path plan and the characteristics of the driver.

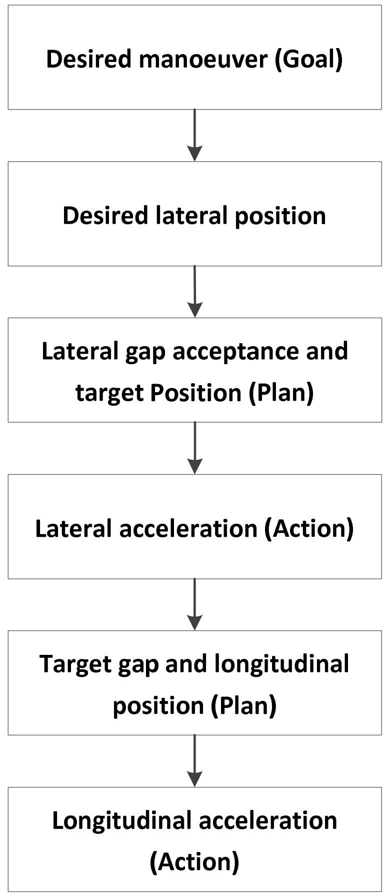


Figure 11 Overall Model Framework for Driver Behavior in Mixed Traffic

The chosen desired manoeuver dictates a desired lateral position, where the driver would like to position the vehicle in order to complete the desired manoeuver. Figure 12 shows possible desired lateral positions for various manoeuvers. In CF and SF, the desired lateral position may be the centerline and the edge, respectively, of the intended lead vehicle. In FB, the desired lateral position may be the middle point between the two leaders. In PS, the desired lateral position may be one that maintains a minimum safe distance the subject vehicle and the closer leader.

In some cases, it may not be feasible for the driver to immediately move to the desired position due to the presence of other neighboring vehicles. Therefore, a target lateral position may be defined, which is the furthest the driver able to move in the direction of the desired position. This position may be determined by applying gap acceptance functions on the available gaps in the direction of the desired lateral movement. This process is shown in Figure 13. Suppose that the subject vehicle (S) decides to follow leader (L), the desired lateral position is therefore directly behind the intended leader. The driver then evaluates the gaps with each one of the vehicles between its current and desired position from nearest to farthest (Gap 1 to Gap 4). The target position is dictated by the first gap to be rejected. For example, if gap 3 is rejected the target position would be dictated by the position of this vehicle and a safe lateral clearance. Gap acceptance decisions depend on the magnitude of the available gaps, the relative speed of the two vehicles, the types of vehicles and the urgency of the lateral movement.

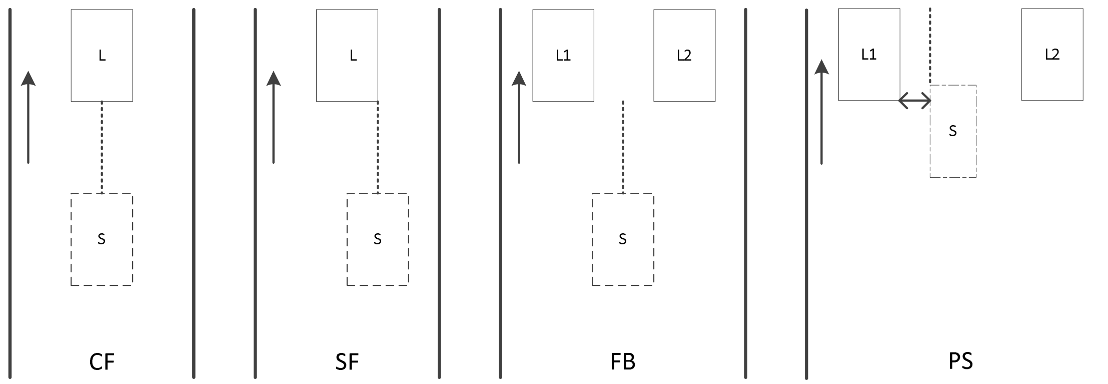


Figure 12 Desired Lateral Positions

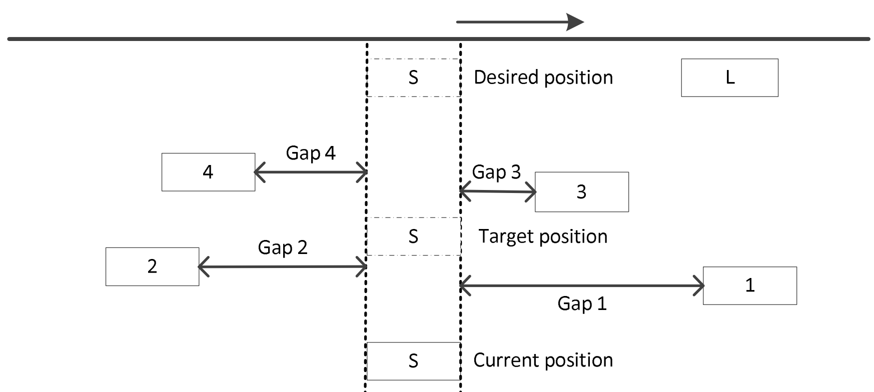


Figure 13 Lateral Gap Acceptance and Target Position

For the lateral acceleration, as well as all other acceleration behaviors, it may be assumed that the driver reacts to different stimuli depending on the driving regime. For example, in lateral acceleration, the driver may react to the distance between the current position and the target position. In longitudinal acceleration, the driver may react to the leader relative speed in CF and SF, the relative speeds of both leaders in FB and PS.

This framework outline allows to form integrated models for driving behavior that capture both longitudinal and lateral movement of vehicles and hence, inter-dependencies among them and different behaviors such as staggered following, vehicle following between two vehicles and passing.

6.0 CONCLUSIONS

This paper reviews the state-of-the-art in driver behavior models under mixed traffic conditions: Longitudinal acceleration, lateral shift and gap acceptance models. Mixed traffic is characterized by a wide range of vehicle types and lack of lane discipline. As a result, there are driving behaviors that are specific to mixed traffic streams, such as staggered following, following between two vehicles, and passing and lateral shifts. There have been many attempts to model these behaviors separately. This paper outlines an integrated model framework for the two-dimensional movement of vehicles that has the potential to capture inter-dependencies in the movements. A major obstacle to development of mixed traffic driving models is the limited availability of trajectory data that is needed for estimation of their parameters.

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