Optimal Dynamic Tolls for Managed Lanes

Tomer Toledo, Omar Mansour, and Jack Haddad

This paper presents a real-time simulation-based control framework to determine dynamic toll rates to optimize an operator's objective subject to various operational and contractual constraints, such as smooth toll rate changes and maintenance of prescribed levels of service on the toll lane. The toll-setting system incorporates models to predict both the vehicle arrival process upstream of the toll lane facility and drivers' choice whether to use the toll lanes as a function of the toll rate and travel times presented to drivers within the information system. A macroscopic traffic simulation model is used to predict the flow conditions within the prediction horizon. The travel times provided to users as information and the travel times predicted by the traffic flow model are iterated until consistency between them is obtained. The whole process is embedded within an optimization algorithm that sets tolls to optimize a given objective function. Several case studies demonstrate the use of this framework and its potential to provide useful toll settings.

Managed lanes, such as toll or high-occupancy-toll (HOT) lanes, are efficient means to mitigate traffic congestion through management of travel demand. "Dynamic tolling" refers to the situation in which tolls vary on the basis of traffic conditions and allow operators to control the utilization of the toll facility in response to changing traffic and demand conditions.

The tolling scenario considered in this research is shown in Figure 1. A freeway section consists of free and toll lanes. Vehicles approaching the section receive information through variable message signs on the current toll rate. They may also receive travel time information on the toll lane, on the free lanes, or both. Drivers then choose to use either the toll or the free lanes. The toll-setting problem addresses the toll rate chosen by the road operator, which may change at regular control steps (e.g., every 5 min).

Early works on this problem used static toll strategies, that is, constant or varying on the basis of a priori definitions, such as time of day. Lindsey and Verhoef (*1*) and Li and Govind (*2*) developed optimal constant toll rates by assuming time-independent demands and traffic conditions and by taking into account the effect of the toll rates on drivers' selection between the toll and free alternatives.

Recently, methods to determine dynamic tolls in real-time have also been proposed. These typically sought to maintain free-flow conditions on the toll lane while maximizing the throughput of the freeway. Yin and Lou developed and compared two such strategies (*3*). The first was based on feedback control. In that strategy, the toll rate at a given time step depended on the toll rate at the previous step and the occupancy at a bottleneck downstream of the end of the toll lane. The second was based on reactive self-learning. This approach formulated a discrete choice model to capture drivers' decisions whether to use the HOT lane. The model was based on drivers' willingness to pay for travel time savings, which is learned over time from observations on traffic data and toll road use. The toll rate itself was determined as a function of the approaching demand, the estimated travel times, and the willingness to pay. Simulation results demonstrated the effectiveness of the two approaches, in particular the self-learning approach. Lou et al. (*4*) and Lou (*5*) expanded the reactive self-learning approach by using a macroscopic traffic flow model to create a more realistic representation of traffic dynamics. This model captured the effects on travel times and throughput of lane changes that take place at the upstream end of the toll lane. The toll rate in each control step was determined by solving an explicit nonlinear optimization problem. Traffic state estimation, demand prediction, and learning of willingness to pay were also introduced in the model.

The methods just described are based on the maximization of the throughput or the satisfaction of constraints on the speed or density in the toll lane. But toll lane operators may be more interested in maximizing other objectives, such as revenue or social welfare. In addition, these methods do not ensure that the travel time information provided to travelers is consistent with the travel times that they experience in the system. This paper presents a framework for a model-predictive toll-setting system that extends previous ones in the following ways. First, it allows operators the flexibility to define explicitly any objective function that they may wish to optimize. Second, it accounts for the effect of not only the toll rate information but the travel time information that is provided to the drivers on their choices. Thus, the framework may be used to ensure that travel time information provided to drivers will be consistent (i.e., that it will agree with the travel time predictions based on the responses of drivers to information they receive) and to evaluate the impact of various ways to present and display the information.

The rest of this paper is organized as follows. The next section presents the overall framework and components of the toll-setting system. Then a case study is used to demonstrate the operations of the toll-setting system. That is followed by the results from the case study. The final section discusses the results and potential enhancements and extensions to the system.

MODEL DESCRIPTION

Overall Framework

The system is designed to reside at the control center for the tolled facility and to operate in real time. The overall framework of the toll-setting system is shown in Figure 2. The optimization process shown in the figure is run at the beginning of each control step *t* to

Faculty of Civil and Environmental Engineering, Technion—Israel Institute of Technology, Haifa 32000, Israel. Corresponding author: O. Mansour, omar@technion.ac.il.

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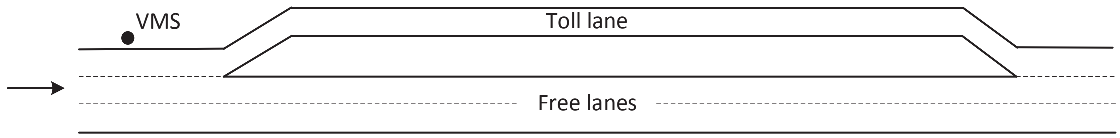

FIGURE 1 Road section with both free and toll lanes (VMS = variable message sign).

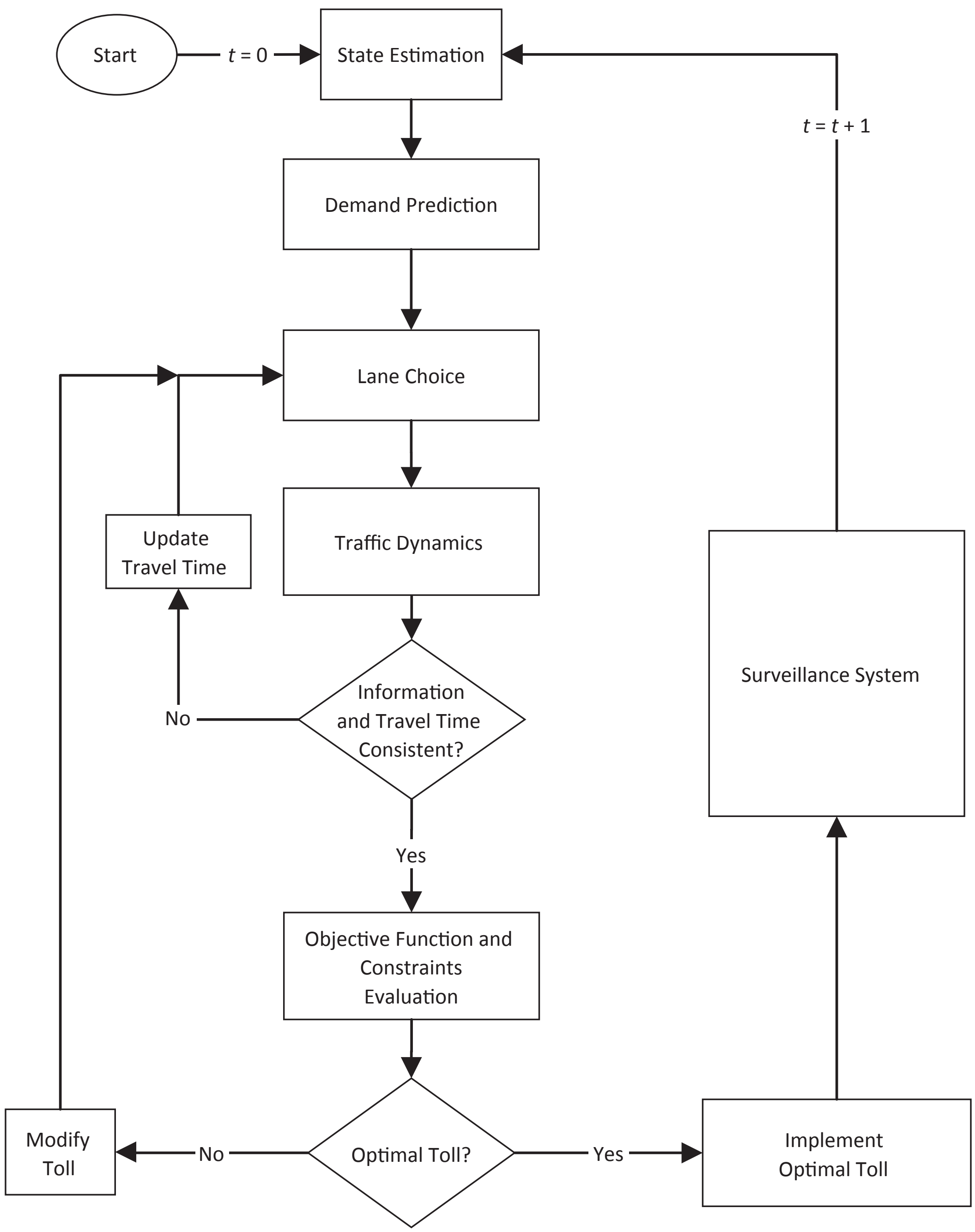

FIGURE 2 Framework of toll-setting system.

determine the toll rate for that interval. A "control step" is defined as the time between each update of the toll rate (typically, 3 to 5 min). The system first uses available traffic measurements and historical data to estimate the current state of the network (i.e., travel times and densities) and to predict the overall demand of arriving vehicles in the next intervals. Then, an initial value for the toll rate is used within a lane choice model to predict the shares of drivers who will choose the toll and the free lanes. The estimated network state and the predicted demands are used as initial boundary conditions for a traffic flow model to predict flows, speeds, and travel times. If travel time information is provided to the drivers, it is updated according to the predicted values. This update, in turn, affects the prediction of lane choices, which affects traffic flow and so on. Therefore, the process iterates until consistency of the predicted and informed travel times is achieved.

The prediction process is embedded in an optimization algorithm that uses the predicted traffic flow characteristics to calculate the value of the objective function and iteratively to find the optimal toll rate. The optimization process is implemented in a rolling-horizon framework, which is shown in Figure 3. At each time step, the simulation is run for a certain prediction horizon *h.* Within this horizon, optimal toll rates are calculated for *m* control steps that define a control horizon ($m \leq h$). The optimal toll rate for the first step is implemented in the field. When the time for the next step arrives, the process is repeated.

Traffic Dynamics

The presented framework requires a realistic representation of traffic dynamics on both free and toll lanes and the use of a traffic flow model. The traffic model should be capable of estimating the time-dependent traffic states within the prediction horizon and the effect on these states of past and future control actions (i.e., toll rates) and of the demand inputs. The traffic flow model outputs (e.g., flows, speeds, and travel times) are used not only to help calculate the objective function but to evaluate satisfaction of the set of constraints on the toll lane operations (e.g., minimum speed, maximum flow, or travel times). The model predictions provide early warnings of potential problems that may affect toll settings ahead of their predicted occurrence. The traffic dynamic model should have the following main characteristics:

1. Easy implementation and adaptation within the framework,
2. Real-time use capability, and
3. Easy calibration and error handling.

In the current implementation, the cell-transmission model (CTM), a macroscopic first-order traffic flow model is used (*6, 7*). CTM divides the freeway into homogeneous sections (cells) so that vehicles move from an upstream cell to the next downstream cell. CTM simulates the freeway system with a time-paced strategy in which traffic states are updated in every simulation step. The inputs to CTM are these:

1. A vector of the predicted demands for the prediction horizon,
2. A vector of the split ratios for the prediction horizon, and
3. Initial densities along both the free and toll lanes.

The model outputs are flows, densities, speeds, and travel times in each cell and at every simulation step within the prediction horizon. From these outputs, the values used by the toll-setting system may be extracted:

1. Predicted travel times for both the toll and free lanes within the prediction horizon,
2. Predicted traffic states at the beginning of the next control step, and
3. Predicted number of vehicles that will enter the toll lane in each step within the prediction horizon.

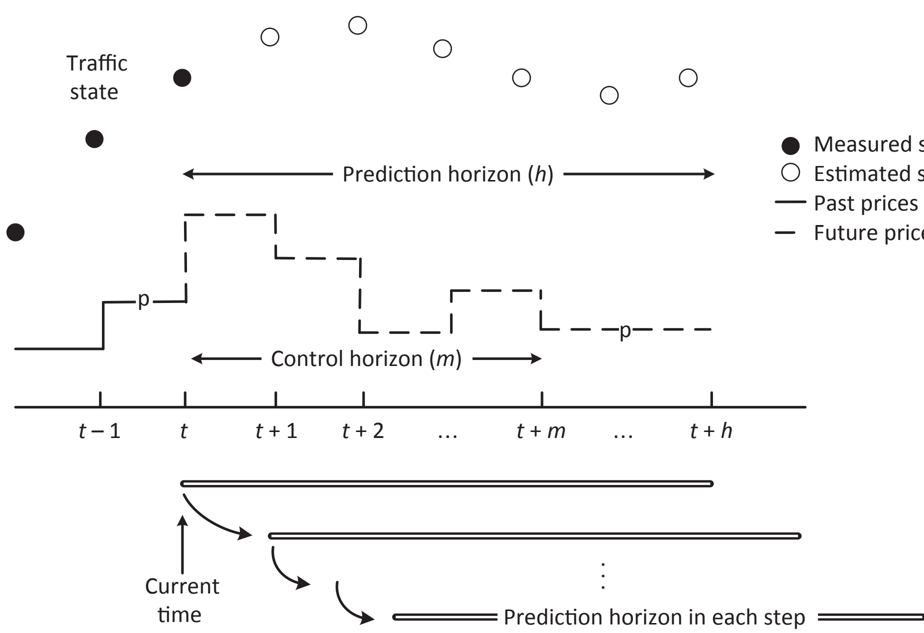

FIGURE 3 Framework of rolling time horizon.

Lane Choice

The ability to predict travelers' choices of whether to use the toll lane is central to the entire process of setting optimal tolls, which are ultimately designed to affect traffic conditions through this choice.

In the wider context of route choices many previous studies [e.g., Bonsall (*8*) and Ramming (*9*)] have shown that the factors affecting this choice include travel times and delays, travel distances, tolls, physical characteristics of the route (e.g., types of roads or presence of intersections), safety, and security. Information provided to the drivers in real time about these factors, in particular traffic conditions and tolls, also have been shown to affect route choices. Both the content and the presentation of the information have been shown to affect route choices [e.g., Wardman et al. (*10*), Hidas and Awadalla (*11*), Peeta and Ramos (*12*), Erke et al. (*13*), and Zheng and Levinson (*14*)].

Somewhat surprisingly, the choice between toll and free lanes within a highway has not received much attention in the literature. Janson and Levinson evaluated the values of time that are reflected in this choice (*15*). They found that the willingness to pay for travel time savings in this context is substantially higher than that commonly found in other route choice scenarios.

The scenario assumed in the current study is that drivers receive information when they approach the entrance to the toll lanes. This information included the current toll rate and may have also included information on the travel times and level of congestion. This information may be related only to the managed lane (as implemented, for example, on the I-91 express lanes between Orange and Riverside counties in California), only to the free lanes (e.g., Highway 1 fast lane in Israel), or to both. In other cases, no information at all was provided and drivers were left to rely only on their own perceptions (e.g. SR-167 HOT lanes in the State of Washington). The current implementation assumed that drivers were provided with full travel time predictions on both the free and toll lanes. Furthermore, for simplicity, it assumed that the lane choices were affected only by these two variables. The lane choice probability, which defines the split ratio that was introduced earlier, was modeled by using a binary logit model and given by

$$\beta_t = \frac{1}{1+\exp\left[-\left(\alpha_0+\alpha_1\left(\mathrm{tt}_t^T-\mathrm{tt}_t^F\right)+\alpha_2 p_t\right)\right]} \tag{1}$$

where

β_t = probability of choosing the toll lanes in simulation step t,
tt_t^T and tt_t^F = expected travel times on the toll and free lanes, respectively,
p_t = toll rate, and
α_0, α_1, and α_2 = scalar parameters.

In the proposed framework, at each time step, this model is applied to the h steps within the prediction horizon on the basis of corresponding vectors of expected travel times and toll rates. The output of this model is a vector of split ratios of drivers who will choose the toll lanes in these intervals.

Demand Prediction

The proposed model deals with traffic states within a prediction horizon. Therefore, it needs predictions of the future incoming flows to the traffic network within the prediction horizon. The demand prediction model must be fast for online use, reliable, and capable of coping with modest amounts of data. Statistical time series techniques are well suited for this purpose. Williams and Hoel showed that a seasonal autoregressive integrated moving average model can be used for short-term predictions of traffic flow in freeways (*16*). In the current implementation, this model was adopted to predict the traffic inflow at the entrance of the freeway section within the prediction horizon. The forecast used time series data on these inflows from previous seasons (e.g., weeks) and earlier time intervals on the same day. Mathematically, the prediction is given by

$$\begin{aligned}\hat{q}_t &= q_{t-H}+\varphi\left(q_{t-1}-q_{t-1-H}\right)-\theta\left(q_{t-1}-\hat{q}_{t-1}\right)\\ &\quad -\Theta\left(q_{t-H}-\hat{q}_{t-H}\right)+\theta\Theta\left(q_{t-1-H}-\hat{q}_{t-1-H}\right)\end{aligned} \tag{2}$$

where

$\hat{q}_t$ and q_t = predicted and measured incoming flows, respectively, in step t,
θ, φ, and Θ = model parameters, and
H = number of intervals in a season.

Objective Function and Constraints Evaluation

The demand estimates and the traffic flow models are used within an optimization framework. Toll lane operators may adopt several objective functions, such as maximizing revenue, social welfare, or throughput. The objective may also be subjected to various operational and contractual constraints, such as requiring smooth transitions in toll rates and enforcing lower and upper bounds on toll rates, bounding the densities, or requiring minimum speeds or minimum flows during certain periods of the day on the toll lanes.

The decision variables in this optimization problem are the m toll rates for the steps within the control horizon. The current implementation assumed that, after the end of the control horizon (step $t+m$ and onward until the end of the prediction horizon in step $t+h-1$), the toll rate was held constant at its last value (from step $t+m-1$).

State Estimation and Learning

To achieve better accuracy, the toll-setting system was designed to be able to use measurements from traffic sensors that may be available in the system. These data were used to correct errors in the initial states of the traffic system that served as inputs to the traffic dynamics model and in predicting future inflows to the system (Equation 2). The measurements may also be used to learn and refine values of the parameters within the various model components (e.g., the demand prediction model described earlier, which uses sensor count measurements to adjust earlier demand estimates). Similar methods may be used to calibrate online the parameters of the lane choice and traffic dynamics models when new measurements are received (*17–20*).

DEMONSTRATION CASE STUDY

To demonstrate the use of the toll-setting framework, several simulation experiments were conducted with the toll-setting system. The toll road facility used in the case study, shown in Figure 4, is based on the one for Highway 1 in Israel. This is a four-lane, 14-km freeway

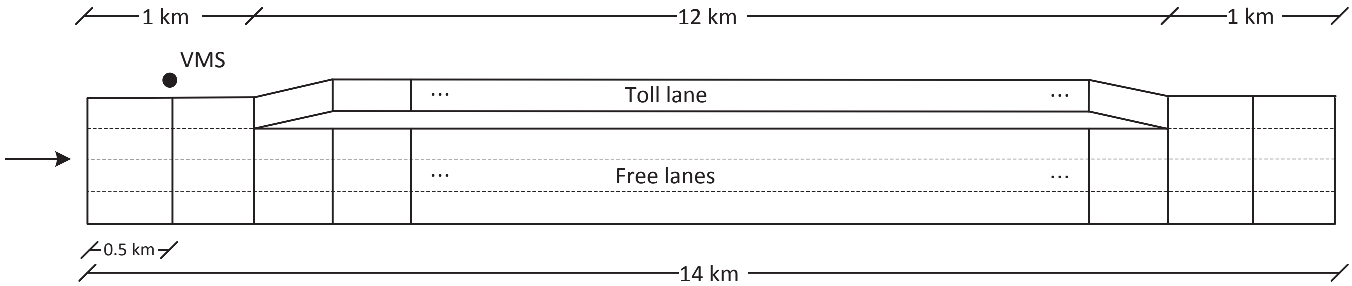

FIGURE 4 Scheme of road section used in case study.

section. A toll information sign is located at the upstream end of the section, which is 1 km upstream of the entrance to the toll lane. The main section of the highway has three free lanes and a single toll lane. This section is 12 km long. At the downstream end, the toll and free lanes merge again and all four lanes are free.

In the CTM, this network is represented with cells that are 0.5 km long. The traffic states are updated every 15 s (simulation step). The traffic fundamental diagrams for cells in regular and bottleneck conditions are shown in Figure 5. Bottleneck cells (e.g., during incident conditions) were constructed to have 50% of the capacity of a regular cell. The free-flow speed is 90 and 45 km/h for the regular and bottleneck cells, respectively.

Two profiles for the total flow entering the section were considered in the experiments. These are shown in Figure 6. They differ in the duration for which the peak demand exists.

The parameters of the lane choice model were set such that when the travel times on the toll and free lanes are equal, and the toll rate is zero, drivers will split equally among all lanes. Other assumptions were that the value of time was 50 new Israeli shekels (NIS) (1 NIS = \$0.26 in January 2017) and that the use of the toll lane when the travel times were equal was less than 5% when the toll was above 20 NIS. The values of parameters of the lane choice model that satisfy these assumptions are $\alpha_0 = -1.099$, $\alpha_1 = -4.149$, and $\alpha_2 = -0.083$, respectively. Tolls were assumed to change in intervals of 5 min and increments of 3 NIS. Another assumption was that the toll road operator was required to keep the travel time on the toll lane to less than 20 min. If this constraint was violated during some interval, the entire revenue collected within that interval was surrendered to the state. These constraints were loosely based on the ones implemented in Highway 1 in Israel. The objective function used in the case study was to maximize operator revenue over the prediction horizon while taking into account the possibility of lost revenue as just described. In the case study, the prediction and control horizons were both set to three steps (15 min). Thus, the objective function is given by

$$\max_{p} \sum_{s=t}^{t+h-1} n_s^T p_s \delta_s^{\mathrm{tt}^T \le 20} \tag{3}$$

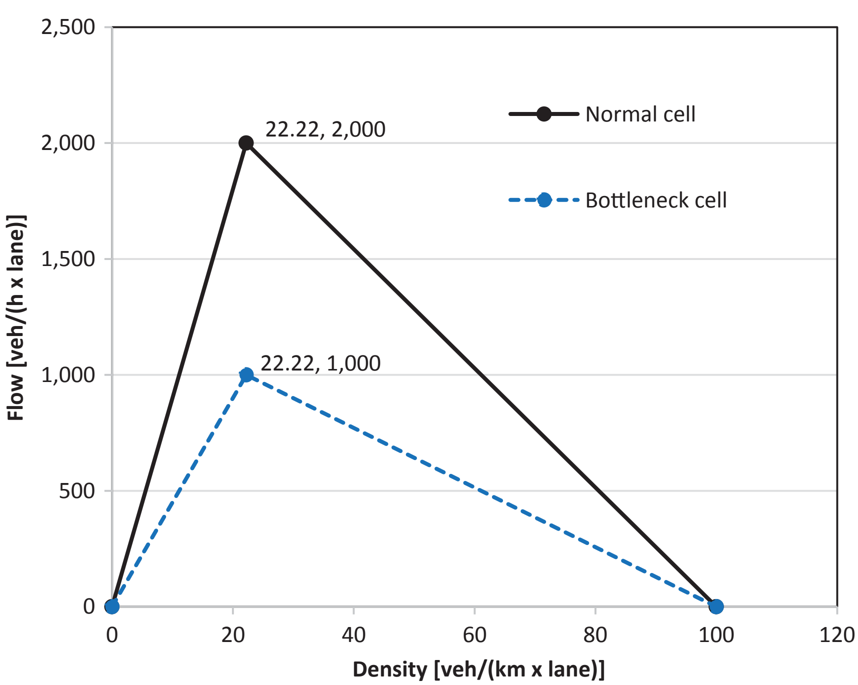

FIGURE 5 CTM fundamental diagrams for regular and bottleneck cells (veh = vehicles).

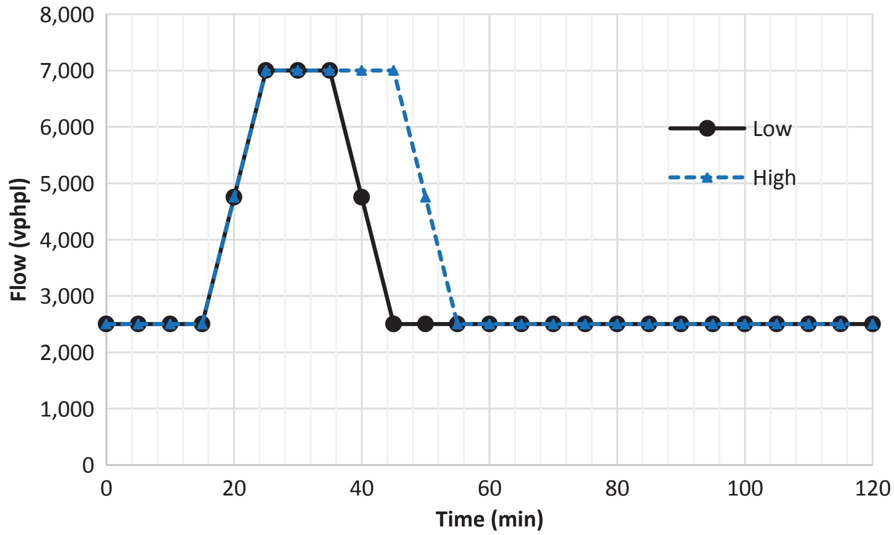

FIGURE 6 Standard and extended demand profiles (vphpl = vehicles per hour per lane).

where

n_s^T = number of vehicles that entered the toll lane during interval s,
p_s = toll rate at interval s, and
$\delta_s^{tt^T \leq 20}$ = indicator variable (= 1 if travel time on toll lane for vehicles entering it during interval s is less than 20 min and = 0 otherwise).

Experiments with System

Several scenarios were used to evaluate the performance of the toll-setting system: Scenario 1 used the standard demand profile shown in Figure 6, with the full capacity of all cells in the network. This scenario was used to examine the effect of different initial toll rates on the dynamics of the tolling and traffic flow produced by the system. Scenario 2 added a bottleneck cell, with reduced capacity at the merging of the free and toll lanes at the downstream end. Scenario 3 used the same bottleneck characteristics but with an extended demand profile. In each of these scenarios, the CTM model itself was used to produce the measurements of flows, travel times, and densities that were used within the toll-setting system. Thus, the assumption was made that the model predictions were error free. Scenarios 4 and 5 evaluated the effect of this idealistic assumption on the model performance through introduction of errors in the prediction of the demand and in the estimation of the current network state.

Scenario 1. Effect of Initial Conditions

The Scenario 1 experiment examined the effect of different initial toll rates on the dynamics of the toll rate and traffic flow. The results presented in Figure 7 show the evolution of the tolls, with two initial toll values: 21 NIS (Figure 7*a*) and 6 NIS (Figure 7*b*). In both cases, the toll converged in no more than four time steps to the steady state toll value of 12 NIS. Evolution of the lane choice probabilities was consistent with that of the toll rates and also converged to an equilibrium value of $\beta = 0.11$ (Figure 7, *c* and *d*).

Scenario 2. Base Demand, Reduced Merge Capacity

The Scenario 2 experiment demonstrated the response of the system to the formation of congestion on the free lanes. The results are presented in Figure 8. The travel time on both the free and toll lanes increased (Figure 8*a*) as congestion built upstream of the bottleneck cell when the total inflow increased to 7,000 vehicles per hour (Figure 8*b*). As a result, the toll increased to avoid congestion on the toll lanes and loss of the revenue to the operator (Figure 8*c*). The travel time on the free lanes peaked after 35 min and then gradually decreased when the queue dissipated after the demand started to decrease. The toll rate followed a similar trend. It initially increased from its initial value of 9 NIS to 18 NIS. It decreased to 15 NIS when the travel time on the free lanes started to decrease but stayed at that level for seven control steps (35 min), until the queue on the free lanes dissipated completely. At that point, the toll decreased to the steady state value of 12 NIS. During peak demand, the flow on the toll lane increased as a result of both the increase in demand and an increase in the fraction of drivers choosing the toll lane, but the travel time constraint was never violated (Figure 8*d*).

Scenario 3. Extended Peak Demand

In the Scenario 3 experiment, the same setup as in the previous one was used but with the extended demand profile. The results are presented in Figure 9. The prolonged peak demand resulted in higher tolls, which reached 24 NIS (Figure 9*a*). Again, the relationship between the travel time on the free lanes and the toll rate was evident. The results also exemplified the effect of the use of future predicted demand. Travel times at the intervals starting at Minute 40 and at Minute 55 were similar on both lanes (Figure 9*b*). But the toll rates were different. The figure for the split-cell outflow graph shows that the demand decreased significantly between these two periods (Figure 9*c*). Therefore, in the later period, the toll rate was lower and in turn encouraged drivers to choose the toll lane (Figure 9*d*).

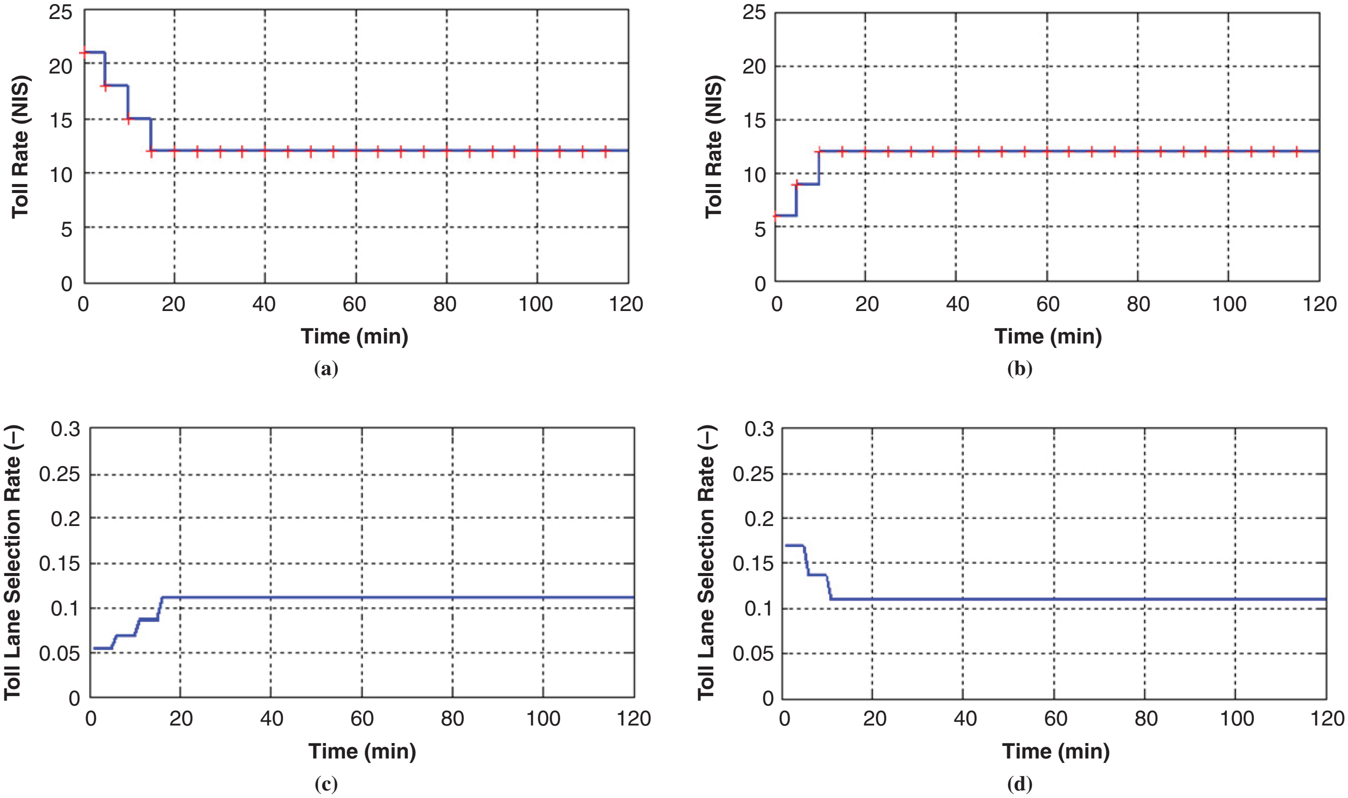

FIGURE 7 Tolls and lane choice probabilities with different initial toll values: (*a*) toll rate evolution for 21 NIS initial toll, (*b*) toll rate evolution for 6 NIS initial toll, (*c*) lane choice for 21 NIS initial toll, and (*d*) lane choice for 6 NIS initial toll.

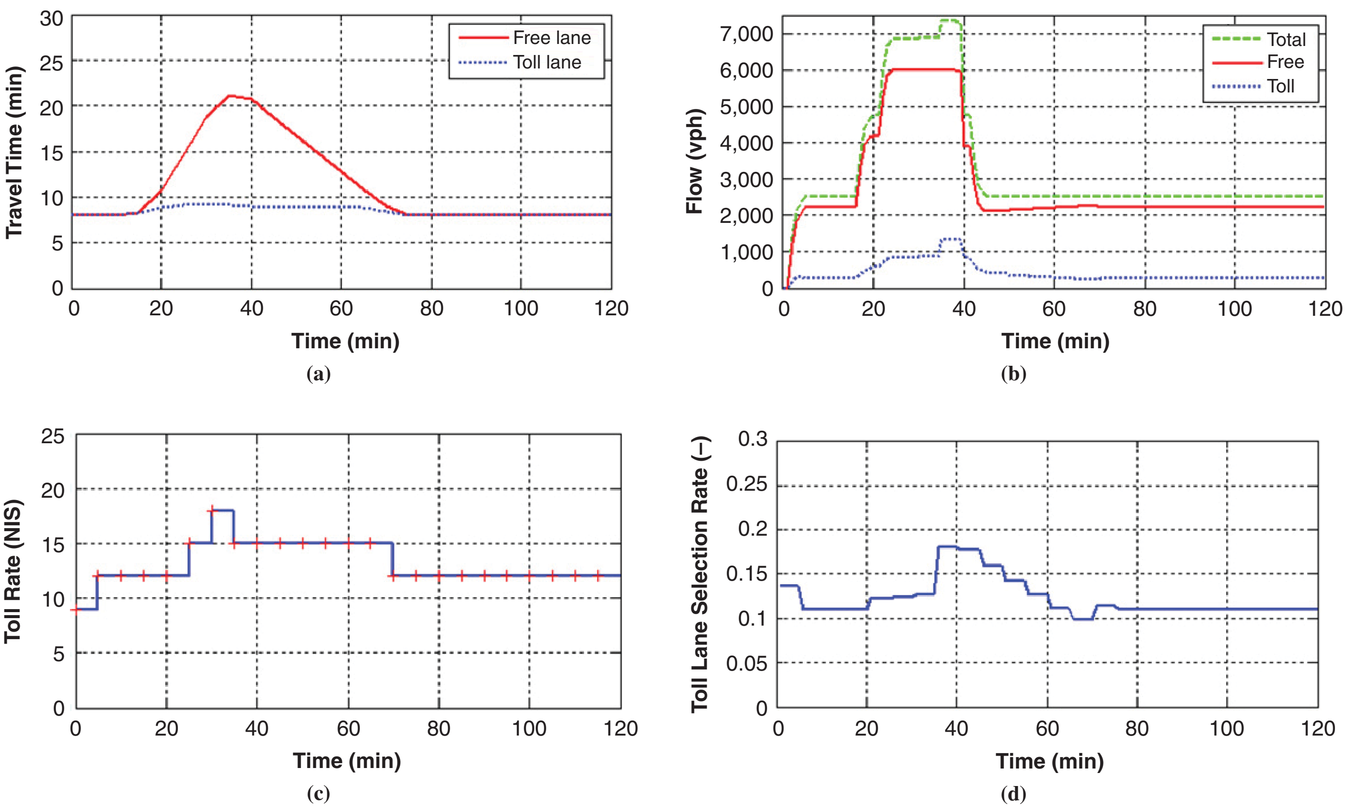

FIGURE 8 Results of Scenario 2: (*a*) estimated travel time, (*b*) split-cell flow, (*c*) toll rate, and (*d*) lane choice.

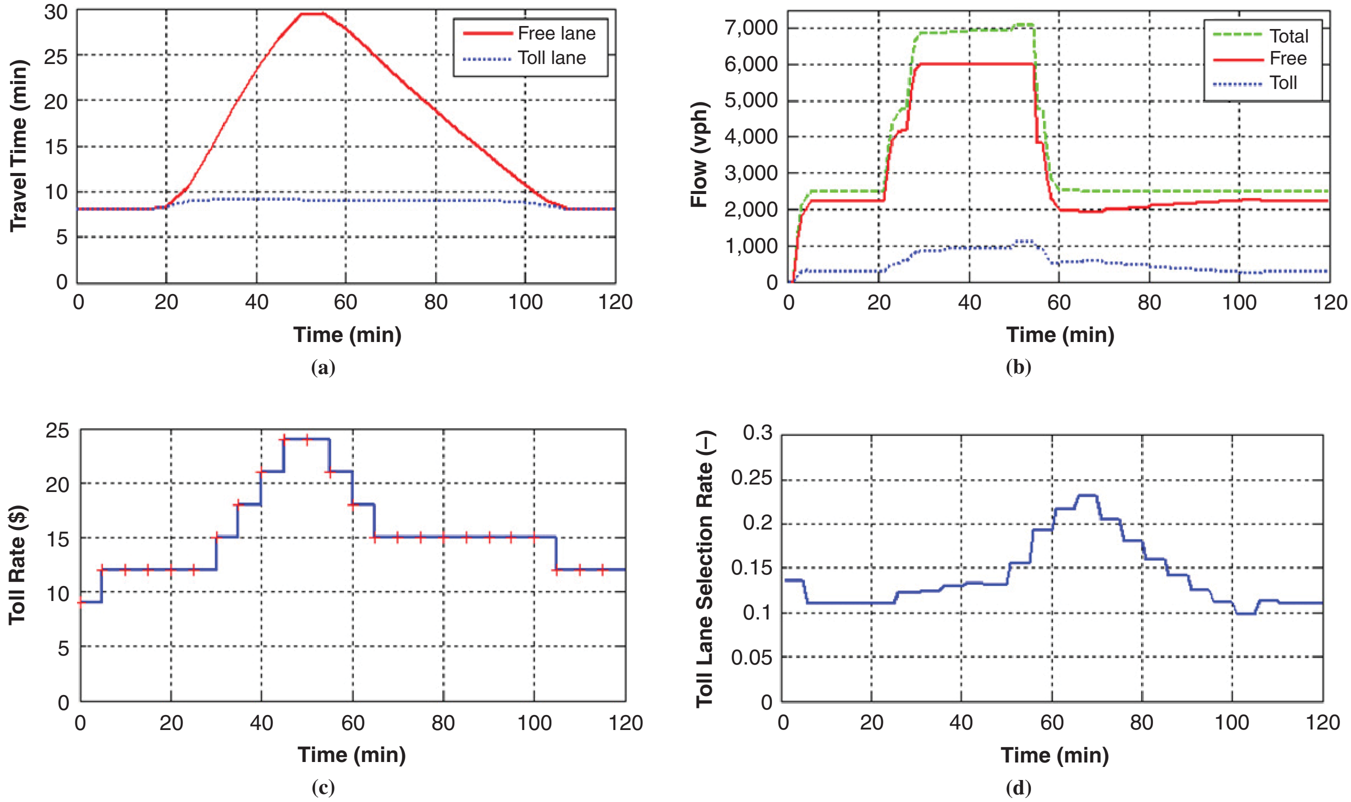

FIGURE 9 Results of Scenario 3: (*a*) estimated travel time, (*b*) split-cell flow, (*c*) toll rate, and (*d*) lane choice.

Scenario 4. Effect of Prediction Errors

Two main sources of error in the model were the predictions of traffic demand entering the corridor and the estimates of current traffic conditions (densities) that were used as initial values for the CTM model. By using the setup of Scenario 3, artificial random errors were implemented on one of these two models, while the other model was kept error free. In each case, the standard deviation of the error was expressed as a percentage of the "true" mean value. Figures 10 and 11 present the effect of the prediction error in the demand and in the state, respectively, on the revenue. The results were normalized against the revenue obtained with error-free models. The reported

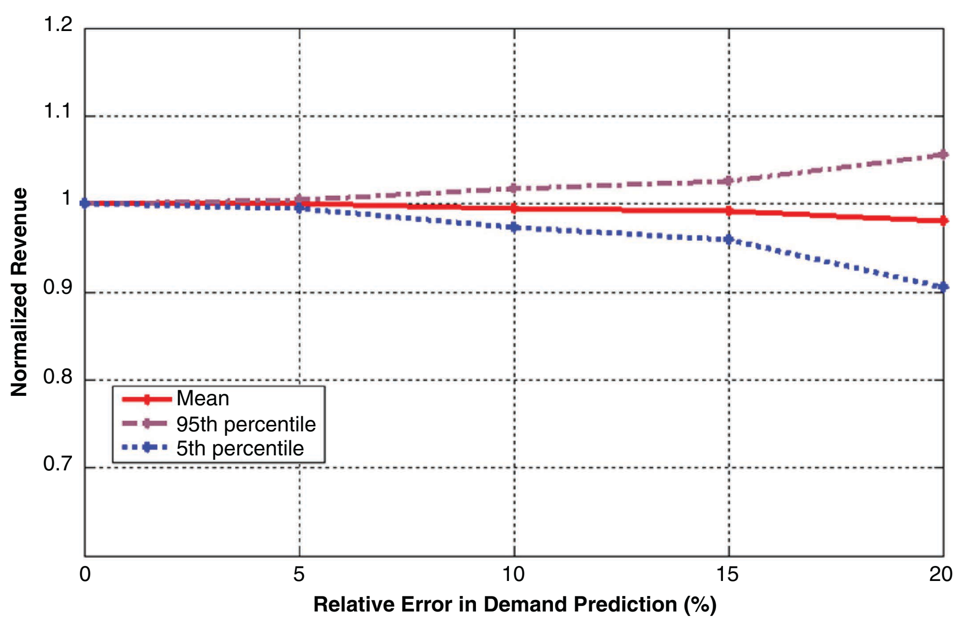

FIGURE 10 Effect of demand prediction errors on the revenue.

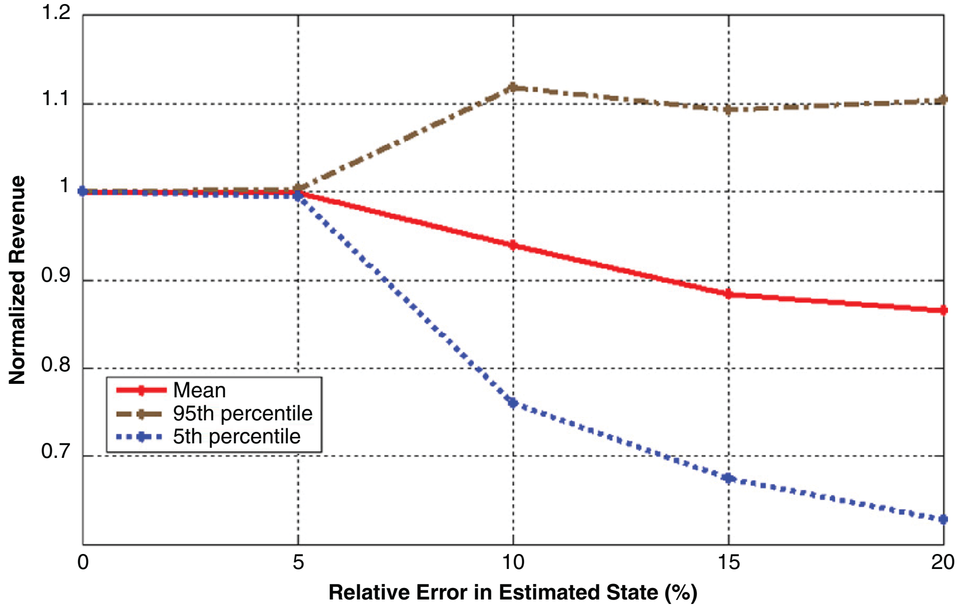

FIGURE 11 Effect of state estimation errors on the revenue.

results were based on 20 model runs in each case. The figures show the mean as well as the 5th and 95th percentiles, which correspond to the mean ±2 standard deviations of the revenue. As expected, the mean revenue decreased with an increase in the errors of both the demand prediction and the state estimation. However, the reduction attributable to demand errors was slight, reaching only 3% when the prediction error was 20%. With the state estimation, the loss of revenue was larger, up to 13%. In specific cases, modeling errors may lead to increased revenue. However, such errors will also result in provision of unreliable information to the drivers, which, in the long run, will lead to loss of trust by the drivers and consequently to loss of the ability to affect use patterns of the toll lane.

SUMMARY

This paper presents a real-time simulation-based control framework to determine dynamic toll rates to optimize an operator's objective subject to various operational and contractual constraints. The toll-setting system incorporated models to predict both the vehicle arrival process upstream of the toll lane facility and drivers' choice whether to use the toll lanes as functions of the toll rate and travel times presented to drivers within the information system. A macroscopic traffic simulation model was used to predict the flow conditions within the prediction horizon. The travel times provided to users as information and the ones predicted by the traffic flow model were iterated until consistency between them was obtained. The whole process was embedded within an optimization algorithm that set tolls to optimize a given objective function. Several case studies demonstrated the use of this framework and its potential to provide useful toll settings.

The system presented in this paper was applied to a road section with a single entrance and single exit. However, increasingly, networks of toll facilities are being built. Thus, a natural extension is to adapt the toll-setting system to multiple entry and exit points. This adaptation requires the use of efficient optimization algorithms that would support a larger number of variables and constraints. It also calls for clearer understanding of users' route choice and the effect of the various operational and contractual constraints on the tolling system on the objective function.

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